FIXED INCOME RISK ENGINE

Initial Margins

Methodological notes

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1 Introduction

The purpose of this document is to describe the steps for computing the *Initial Margins Expected Shortfall* of the portfolio subject to margining.

The securities to which the process described in this document is applied are the following:

1) Italian government bonds:
2) Spanish government bonds:
3) Irish government bonds:
4) Portuguese government bonds:

Therefore, corporate bonds and government bonds that are not part of the MTS GC-EXTRA basket are currently excluded from the application of the *Expected Shortfall* computation (the current *Initial Margins* computation methodology based on SPAN-like margin intervals therefore remaining in force).

Cash-flow mapping

First, the cash flows of each security belonging to the portfolio subject to margining are assigned to the proper risk factors. In particular, the current market value of each security is split onto its cash flows (each having its own duration), which are subsequently mapped onto the proper tenors of the *sovereign zero-coupon (ZC) spot curve* which the security refers to (e.g. Italian *ZC curve* for Italian government bonds).

Cash-flow mapping is applied to a subset of the Clearing Member's portfolio composed only of *cash* and *repo* positions, as for *forward starting repo* positions, as illustrated in the *Mark-to-Market Margins* module, the exposure to the bond price movements is both long and short, thus resulting in a net 0 exposure.

Price scenarios

Then, *price variation scenarios*, both *unscaled* and EWMA-*scaled*, are computed for the *ZC curves* tenors impacted. These scenarios will be employed in the revaluation of the margined portfolio.

In particular, the *scaling* methodology is based on the introduction of a so-called *smoothing factor* (model parameter), through which it is possible to differently weigh observations of the time series based on current volatility regime. The scaling process consists of the following steps:

1) retrieving *rate* time series of the tenors of the *ZC curves*;
2) converting *rate* time series into *price* time series;
3) computing (*unscaled*) *relative price returns*;
4) computing *EWMA volatilities*;

5) computing *scaled relative price returns*;
6) defining *unscaled price scenarios*;
7) defining *scaled price scenarios*.

Expected Shortfall

Once the (market value of the) cash flows of the portfolio subject to margining have been mapped onto the proper *ZC curve* tenors and the set of *scaled* and *unscaled price scenarios* for these tenors has been defined, the portfolio must be revalued in each of its aforementioned cash flows and *price scenarios*. The comparison between the total value of the revalued portfolio and its current market value yields the profit/loss in the specific *price scenario*. Given the chosen *confidence levels* (model parameters), the *Expected Shortfall* of the margined portfolio can be computed.

The *Expected Shortfall* can be *undiversified* or *diversified* depending on whether, in case of a portfolio composed of bonds issued by multiple countries, the benefit of the diversification between issuers undertaken by the Clearing Member is acknowledged or not.

2 Cash-flow mapping

2.1 Coupon stream definition

Preliminary to the cash-flow mapping procedure is the determination of the coupon stream of each of the securities belonging to the categories listed in the previous paragraph. Particular procedures must be applied to floating-rate securities (*floaters* - i.e. Italian *CCTeu*) and inflation-linked securities (*linkers* - i.e. Italian *BTP Italia* and *BTP€i* and Spanish *linkers*), as described in the following sub-paragraph.

2.1.1 Floaters and linkers

Input data needed

In order to correctly deal with *floaters* and *linkers* (whether European or Italian inflation-linked) the input data needed are:

1) *6M Euribor zero-coupon spot curve* at evaluation date;
2) *6M Euribor* time series;
3) *Zero-coupon spot European inflation curve* at evaluation date;
4) *Zero-coupon spot Italian inflation curve* at evaluation date;
5) *European ex-tobacco CPI (CPTFEMU)* time series;
6) *Italian ex-tobacco CPI (FOI)* time series.

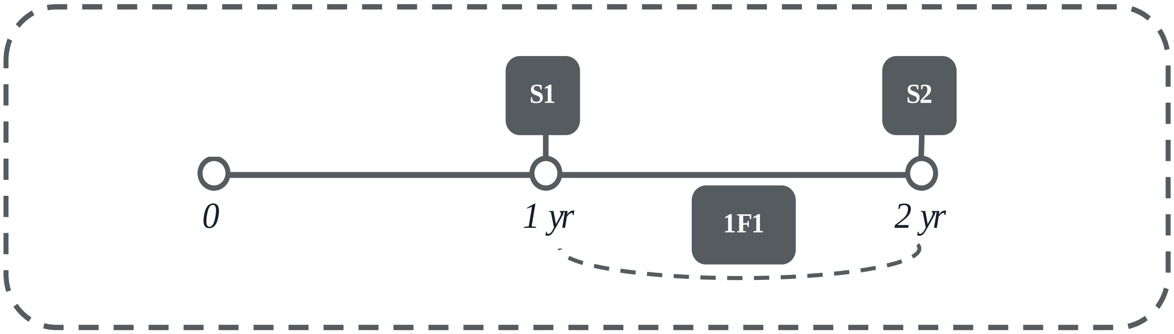
Building the 6M Euribor ZC forward curve at evaluation date for CCTeus

*CCTeu*s are Italian government bonds whose coupons are indexed to the *6M Euribor*. In order to define the coupon stream it is therefore necessary to calculate the future value of the underlying rate at the various *reset dates*. The *reset date* is the day at which the *6M Euribor* employed for defining a given coupon is set. The *reset date* for the current coupon is defined as the last coupon date - 2 working days.

To this aim, the *6M Euribor ZC spot curve* is essential, the *forward rates* being implied by the term structure.

Given a generic term structure, the following relation indeed holds:

Figure 2-1: ***Spot*** **and** ***forward rates***



Assuming you want to invest €1 at 0 for 2 years there are two different options:

1) directly invest for 2 years at *spot rate* S2;
2) invest for 1 year at *spot rate* S1 and then reinvest the amount obtained for 1 year more at *forward rate* 1F1, i.e. the *rate* applied to financial operations that start in 1 year and end in 2 years.

Absence of arbitrage condition implies that the two investment options described above must be equivalent:

(1) $(1+ S2)^2 = (1+S1) * (1+1F1)$,

which in turn implies:

(2) $1F1 = (1+S2)^2 / (1 + S1) - 1$.

Each specific *spot curve* therefore implies a corresponding *forward curve*.

As far as *CCTeu*s are concerned, in order to calculate the future coupon values, thus defining the coupon stream, it is necessary to start from the *6M Euribor ZC spot curve*.

Intermediate, unavailable tenors can be obtained linearly interpolating available ones (e.g. spot_rate_210 = spot_rate_180 + (spot_rate_270 - spot_rate_180) * (210 - 180) / (270 - 180)).

Having the *ZC spot curve*, it is possible to proceed with the calculation of the respective *discount factors*:

Table 1: *Discount factors* calculation

Tenor	Rate	Discount factor
1	spot_rate_1	1 / (1 + spot_rate_1 * 1 / 360)
7	spot_rate_7	1 / (1 + spot_rate_7 * 7 / 360)
…	…	…
…	…	…
2700	spot_rate_2700	1 / (1 + spot_rate_2700 * 2700 / 360)

according to the formula $df = \frac{1}{(1 + r * T)}$, with *r*: *annual spot rate* and *T*: *reference tenor of the spot rate*, expressed in year fraction (day count convention: *act/360*).

Given the calculated *discount factors*, it is possible to compute the *6M forward discount factors* for each of the curve tenors according to the formula $df_{forward\_t} = \frac{df_{t+6M}}{df_t}$:

Table 2: *Forward discount factors* calculation

Tenor	Rate	Discount factor	6M forward discount factor
1	spot_rate_1	df_1	df_181 / df_1
7	spot_rate_7	df_7	df_187 / df_7
…	…	…	…
180	spot_rate_180	df_180	df_360 / df_180
…	…	…	…
2520	spot_rate_2520	df_2520	df_2700 / df_2520

For each of the calculated *discount factors* the respective *6M forward discount factor* must be computed (with the obvious exception of the *discount factor* corresponding to the last tenor, as we will see below). In detail, for each of them, the respective *6M ahead discount factor* must be identified: if the latter is not directly available among those already computed, it is necessary to compute it by linear interpolation. For example, the *6M forward discount factor* for the *O/N* (*i.e. 1d*) *rate* will be equal to the ratio of the *181d discount factor* (starting tenor = 1 day + 6 months) to the *1d discount factor*. While the latter is directly available, the former must be calculated as linear interpolation between the *180d discount factor* and the *210d discount factor* (df_180 + (df_210 - df_180) * (181 - 180) / (210 - 180)). The *6M forward discount factor* for the *7d rate* will be equal to the ratio of the *187d discount factor* (starting tenor = 7 days + 6 months) to the *7d discount factor*. As in the previous case, the latter is directly available while the former must be calculated as linear interpolation between the *discount factors* within which it falls (again, 180 and 210 days). In order to compute the *180d discount factor* no linear interpolation is instead needed, as the two terms of the ratio are both already available (180 and 360 days).

In the above representation, the calculation of the *6M forward discount factors* ends up at tenor 2520 (corresponding to 7 years), obtained as the ratio of the *2700d discount factor* (7.5 years) to the *2520d discount factor* (7 years).

It is finally possible to proceed with the calculation of the *6M forward rates* for each of the tenors for which the respective *6M forward discount factor* has been computed, according to the formula $\text{forward}_{\text{rate}_t} = \frac{1 - \text{df}_{\text{forward}_t}}{\text{df}_{\text{forward}_t} * \frac{180}{360}}$:

Table 3: *Forward rates* calculation

Tenor	Rate	Discount factor	6M forward discount factor	6M forward rate
1	spot_rate_1	df_1	fwd_df_1	(1 – fwd_df_1) / (fwd_df_1 * 180/360)
7	spot_rate_7	df_7	fwd_df_7	(1 – fwd_df_7) / (fwd_df_7 * 180/360)
…	…	…	…	…
180	spot_rate_180	df_180	fwd_df_180	(1 – fwd_df_180) / (fwd_df_180 * 180/360)
…	…	…	…	…
2520	spot_rate_2520	df_2520	fwd_df_2520	(1 – fwd_df_2520) / (fwd_df_2520 * 180/360)

Once built the *6M forward curve* at evaluation date, it is possible to proceed with the definition of the coupon stream of the *CCTeu*s indexed to *6M Euribor*.

Coupon stream definition for CCTeus

In order to define the coupon stream for *CCTeu*s it is necessary to know, in addition to the *forward values* of the underlying rate, the fixed annual *spread* applied to them (*CCTeu*s indeed semiannually pay *6M Euribor + spread*). The sum of these annual rates and the annual *spread* component - multiplied by the number of days between contiguous coupon dates, *i.e.* 6 months - defines the coupons.

Consider the following example:

- ISIN: IT0005104473;
- Maturity date: 15/12/2019;

- Current coupon rate: 0,14;
- Spread: 0,55%;
- Coupon frequency: 6 months;
- Principal: 100;
- Evaluation date: 20/04/2018.

Consider the following *6M Euribor forward curve* at evaluation date:

Tenor	Days	6M Euribor forward rate
O/N	1	-0,00324
1W	7	-0,00318
1M	30	-0,00293
2M	60	-0,00267
3M	90	-0,00238
6M	180	-0,00258
7M	210	-0,00243
8M	240	-0,00229
9M	270	-0,00236
1YR	360	-0,00186
1,5YR	540	0,00183
2YR	720	0,00372

The first step is the determination of the future coupon dates:

Coupon date
15/06/2018
15/12/2018
15/06/2019
15/12/2019

Subsequently, it is necessary to determine the *reset date* of each future coupon date (payment) and compute the relative *time to payment*, *i.e.* the number of days between evaluation date (in the example, 20/04/2018) and each *reset date*:

Coupon date	Reset date	Time to payment (days)
15/06/2018	13/12/2017	past
15/12/2018	13/06/2018	54
15/06/2019	13/12/2018	237
15/12/2019	13/06/2019	419

For each *time to payment*, as shown in the table below, the corresponding *6M Euribor forward rate* is identified (linearly interpolated or directly available looking at the curve in case the *time to payment* coincides with one of its tenors).

Coupon date	Reset date	Time to payment (days)	6M Euribor forward rate
15/06/2018	13/12/2017	past	unnecessary[1]
15/12/2018	13/06/2018	54	-0,002722
15/06/2019	13/12/2018	237	-0,002304
15/12/2019	13/06/2019	419	0,0006505

It is finally possible to proceed with the calculation of each coupon employing the following formula:

$$(4)\ \text{coupon} = \max(0;(\text{forward\_rate}+\text{spread}) * \text{principal} * \frac{\text{coupon\_date-last\_coupon\_date}}{360}).$$

The 15/12/2018 coupon payment will thus be:

$$(-0{,}002722+0{,}0055) * 100 * \frac{15\_12\_2018-15\_06\_2018}{360} = 0{,}14;$$

and so on for the other coupon dates (rounding: 2 decimal places).

At maturity, the payment will be equal to what obtained employing formula (4) plus the repayment of the principal of the bond. The coupon stream of the *CCTeu* of the example can therefore be summarized as in the following table:

Coupon date	Coupon	Notes
15/06/2018	0,14	known, coupon
15/12/2018	0,14	coupon
15/06/2019	0,16	coupon
15/12/2019	100,31	coupon + principal

Building the CPI curves for linkers

In order to define the coupon stream for *linkers*, it is necessary to leverage on the relevant *ZC spot inflation curves* to lengthen the time series of the relevant *CPIs* with their forward values.

First, we need to differentiate between the three types of *linkers* which may be subject to margining:

- BTP Italia – Italian government bonds linked to *Italian ex-tobacco inflation (FOI)*;
- BTP€i – Italian government bonds linked to *European ex-tobacco inflation (CPTFEMU)*;
- Spanish government bonds linked to *European ex-tobacco inflation (CPTFEMU)*.

[1] The 15/06/2018 coupon is already defined and known.

Depending on the type of security subject to margining, it is therefore necessary to proceed with the calculation of different *forward CPI values* (*FOI* or *CPTFEMU*), depending on whether the security is indexed to *Italian* or *European ex-tobacco inflation*. The first step is, as anticipated, the retrieval of the relevant *ZC spot inflation curve*.

Before proceeding with the calculation of the *forward CPI values* it is also necessary to retrieve the *CPI* time series. Both *CPI*s are updated on a monthly basis (usually at mid-month) and with a time lag of 1 month (e.g. at mid-April the March *CPI* value is published). The aforementioned time series will thus have monthly observations and can be represented as follows:

Table 4: *FOI* time series

Date	FOI CPI
31-03-2018	FOI_cpi_0318
28-02-2018	FOI_cpi_0218
31-01-2018	FOI_cpi_0118
31-12-2017	FOI_cpi_1217
…	…

Table 5: *CPTFEMU* time series

Date	CPTFEMU CPI
31-03-2018	CPTFEMU_cpi_0318
28-02-2018	CPTFEMU_cpi_0218
31-01-2018	CPTFEMU_cpi_0118
31-12-2017	CPTFEMU_cpi_1217
…	…

Once the time series are available, it is necessary to identify the *base value* which will be employed in the calculation of the *forward CPI values*. The *base value* is the 3 months-earlier *CPI* value (3 months-time lag) at evaluation date (e.g. if the evaluation date is 04/05/2018 the *base value* will be the 28/02/2018 (February) *CPI* value).

The *forward CPI values* can then be calculated as follows:

Table 6: *Forward CPI - FOI*

Tenor (years)	ZC spot inflation rate	Forward CPI
1	FOI_ZC_1yr_rate	(1 + FOI_ZC_1yr_rate) ^ 1 * *base value*
2	FOI_ZC_2yr_rate	(1 + FOI_ZC_2yr_rate) ^ 2 * *base value*
3	FOI_ZC_3yr_rate	(1 + FOI_ZC_3yr_rate) ^ 3 * *base value*
4	FOI_ZC_4yr_rate	(1 + FOI_ZC_4yr_rate) ^ 4

		* *base value*
5	FOI_ZC_5yr_rate	(1 + FOI_ZC_5yr_rate) ^ 5 * *base value*
6	FOI_ZC_6yr_rate	(1 + FOI_ZC_6yr_rate) ^ 6 * *base value*
7	FOI_ZC_7yr_rate	(1 + FOI_ZC_7yr_rate) ^ 7 * *base value*
8	FOI_ZC_8yr_rate	(1 + FOI_ZC_8yr_rate) ^ 8 * *base value*
9	FOI_ZC_9yr_rate	(1 + FOI_ZC_9yr_rate) ^ 9 * *base value*
10	FOI_ZC_10yr_rate	(1 + FOI_ZC_10yr_rate) ^ 10 * *base value*
12	FOI_ZC_12yr_rate	(1 + FOI_ZC_12yr_rate) ^ 12 * *base value*
15	FOI_ZC_15yr_rate	(1 + FOI_ZC_15yr_rate) ^ 15 * *base value*
20	FOI_ZC_20yr_rate	(1 + FOI_ZC_20yr_rate) ^ 20 * *base value*
25	FOI_ZC_25yr_rate	(1 + FOI_ZC_25yr_rate) ^ 25* *base value*
30	FOI_ZC_30yr_rate	(1 + FOI_ZC_30yr_rate) ^ 30 * *base value*

Table 7: ***Forward CPI - CPTFEMU***

Tenor (years)	ZC spot inflation rate	Forward CPI
1	CPTFEMU_ZC_1yr_rate	(1 + CPTFEMU _ZC_1yr_rate) ^ 1 * *base value*
2	CPTFEMU _ZC_2yr_rate	(1 + CPTFEMU _ZC_2yr_rate) ^ 2 * *base value*
3	CPTFEMU _ZC_3yr_rate	(1 + CPTFEMU _ZC_3yr_rate) ^ 3 * *base value*
4	CPTFEMU _ZC_4yr_rate	(1 + CPTFEMU _ZC_4yr_rate) ^ 4 * *base value*
5	CPTFEMU _ZC_5yr_rate	(1 + CPTFEMU _ZC_5yr_rate) ^ 5 * *base value*
6	CPTFEMU _ZC_6yr_rate	(1 + CPTFEMU _ZC_6yr_rate) ^ 6 * *base value*
7	CPTFEMU _ZC_7yr_rate	(1 + CPTFEMU _ZC_7yr_rate) ^ 7 * *base*

		value
8	CPTFEMU _ZC_8yr_rate	(1 + CPTFEMU _ZC_8yr_rate) ^ 8 * *base value*
9	CPTFEMU _ZC_9yr_rate	(1 + CPTFEMU _ZC_9yr_rate) ^ 9 * *base value*
10	CPTFEMU _ZC_10yr_rate	(1 + CPTFEMU _ZC_10yr_rate) ^ 10 * *base value*
12	CPTFEMU _ZC_12yr_rate	(1 + CPTFEMU _ZC_12yr_rate) ^ 12 * *base value*
15	CPTFEMU _ZC_15yr_rate	(1 + CPTFEMU _ZC_15yr_rate) ^ 15 * *base value*
20	CPTFEMU _ZC_20yr_rate	(1 + CPTFEMU _ZC_20yr_rate) ^ 20 * *base value*
25	CPTFEMU _ZC_25yr_rate	(1 + CPTFEMU _ZC_25yr_rate) ^ 25* *base value*
30	CPTFEMU _ZC_30yr_rate	(1 + CPTFEMU _ZC_30yr_rate) ^ 30 * *base value*

It is then possible to lengthen the observed *CPI* time series with the computed *forward CPI values* to have a complete (observed and forward) time series of *CPI* values, through which it is possible to define the coupon stream for the *linkers*. The two complete time series can be represented as follows (assuming as *base value* that of March 2018):

Table 8: *FOI* complete time series

Date	FOI CPI
31-03-2048	forward_FOI_cpi_30yr
…	…
31-03-2027	forward_FOI_cpi_9yr
31-03-2026	forward_FOI_cpi_8yr
31-03-2025	forward_FOI_cpi_7yr
31-03-2024	forward_FOI_cpi_6yr
31-03-2023	forward_FOI_cpi_5yr
31-03-2022	forward_FOI_cpi_4yr
31-03-2021	forward_FOI_cpi_3yr
31-03-2020	forward_FOI_cpi_2yr
31-03-2019	forward_FOI_cpi_1Yr
31-03-2018	FOI_cpi_0318

28-02-2018	FOI_cpi_0218
31-01-2018	FOI_cpi_0118
31-12-2017	FOI_cpi_1217
…	…
…	…

Table 9: *CPTFEMU* complete time series

Date	CPTFEMU CPI
31-03-2048	forward_CPTFEMU_cpi_30yr
…	…
31-03-2027	forward_CPTFEMU_cpi_9yr
31-03-2026	forward_CPTFEMU_cpi_8yr
31-03-2025	forward_CPTFEMU_cpi_7yr
31-03-2024	forward_CPTFEMU_cpi_6yr
31-03-2023	forward_CPTFEMU_cpi_5yr
31-03-2022	forward_CPTFEMU_cpi_4yr
31-03-2021	forward_CPTFEMU_cpi_3yr
31-03-2020	forward_CPTFEMU_cpi_2yr
31-03-2019	forward_CPTFEMU_cpi_1Yr
31-03-2018	CPTFEMU_cpi_0318
28-02-2018	CPTFEMU_cpi_0218
31-01-2018	CPTFEMU_cpi_0118
31-12-2017	CPTFEMU_cpi_1217
…	…

Coupon stream definition for linkers

Both coupons and principal of *linkers* are revalued on the basis of an *indexation coefficient*, which in turn is a function of the trend of the reference *CPI* over time.

In order to define the coupon stream, the following information are therefore essential:

- issue date;
- maturity date;
- real annual coupon rate;
- coupon frequency;
- principal;
- complete time series of the reference *CPI* (*FOI* or *CPTFEMU*).

For example, consider the following BTP€i at evaluation date: 20/04/2018:

- issue date: 23/04/2014;
- maturity date: 23/04/2020;

- real annual coupon rate: 0,825%;
- coupon frequency: 6 months;
- principal: 100;

and the following complete time series of the reference *CPTFEMU CPI,* obtained as described above:

CPTFEMU	
Date	Mid Price
31/03/2048	172,13
31/03/2043	156,61
31/03/2038	141,95
31/03/2033	128,12
31/03/2030	121,10
31/03/2028	116,84
31/03/2027	114,80
31/03/2026	112,92
31/03/2025	111,10
31/03/2024	109,36
31/03/2023	107,72
31/03/2022	106,28
31/03/2021	104,98
31/03/2020	103,77
31/03/2019	102,54
31/03/2018	101,70
28/02/2018	101,50
31/01/2018	101,50
31/12/2017	101,10
30/11/2017	100,80
31/10/2017	100,90
30/09/2017	101,10
31/08/2017	101,40
31/07/2017	101,00
30/06/2017	101,00
31/05/2017	101,10
30/04/2017	101,30
31/03/2017	101,00
28/02/2017	101,00
31/01/2017	100,60
31/12/2016	100,30
30/11/2016	100,00
31/10/2016	100,00
30/09/2016	100,00
31/08/2016	100,20
31/07/2016	100,00
30/06/2016	99,90
31/05/2016	99,70
30/04/2016	99,60
31/03/2016	99,60
29/02/2016	99,50
31/01/2016	99,70
31/12/2015	99,91
30/11/2015	99,91
31/10/2015	100,09
30/09/2015	99,91
31/08/2015	100,28
31/07/2015	100,09
30/06/2015	100,19
31/05/2015	100,09
30/04/2015	100,00
31/03/2015	99,91
28/02/2015	99,72
31/01/2015	99,44
31/12/2014	99,91
30/11/2014	99,91
31/10/2014	100,09
30/09/2014	100,00
31/08/2014	100,37
31/07/2014	100,19
30/06/2014	100,28
31/05/2014	100,19
30/04/2014	100,28
31/03/2014	100,09
28/02/2014	100,09
31/01/2014	100,19
31/12/2013	100,00

The first step in the definition of the coupon stream for *linkers* is to list the coupon dates (starting from issue date, including also past coupon dates):

Coupon date
23/04/2014
23/10/2014
23/04/2015
23/10/2015
23/04/2016
23/10/2016
23/04/2017
23/10/2017
23/04/2018
23/10/2018
23/04/2019
23/10/2019
23/04/2020

It is then necessary for each of the above dates to identify the relative 2 months-earlier and 3 months-earlier dates, transformed into month-ends:

Coupon date	Coupon date – 2 months	Coupon date – 3 months
23/04/2014	28/02/2014	31/01/2014
23/10/2014	31/08/2014	31/07/2014
23/04/2015	28/02/2015	31/01/2015
23/10/2015	31/08/2015	31/07/2015
23/04/2016	29/02/2016	31/01/2016
23/10/2016	31/08/2016	31/07/2016
23/04/2017	28/02/2017	31/01/2017
23/10/2017	31/08/2017	31/07/2017
23/04/2018	28/02/2018	31/01/2018
23/10/2018	31/08/2018	31/07/2018
23/04/2019	28/02/2019	31/01/2019
23/10/2019	31/08/2019	31/07/2019
23/04/2020	29/02/2020	31/01/2020

For each of the dates identified in the last two columns of the above table, the respective reference *CPI* value must be obtained from its complete time series, linearly interpolating where necessary:

Coupon date	Coupon date – 2 months	Coupon date – 3 months	CPI m-2	CPI m-3
23/04/2014	28/02/2014	31/01/2014	100,0934	100,1867
23/10/2014	31/08/2014	31/07/2014	100,3735	100,1867
23/04/2015	28/02/2015	31/01/2015	99,7199	99,4398

23/10/2015	31/08/2015	31/07/2015	100,2801	100,0934
23/04/2016	29/02/2016	31/01/2016	99,5000	99,7000
23/10/2016	31/08/2016	31/07/2016	100,2000	100,0000
23/04/2017	28/02/2017	31/01/2017	101,0000	100,6000
23/10/2017	31/08/2017	31/07/2017	101,4000	101,0000
23/04/2018	28/02/2018	31/01/2018	101,5000	101,5000
23/10/2018	31/08/2018	31/07/2018	102,0512	101,9800
23/04/2019	28/02/2019	31/01/2019	102,4667	102,4024
23/10/2019	31/08/2019	31/07/2019	103,0520	102,9478
23/04/2020	29/02/2020	31/01/2020	103,6637	103,5662

For each of the rows of the above table an *index number* is computed according to the following formula:

(5) $\text{index\_number} = CPI_{m-3} + \frac{d-1}{dd} * (CPI_{m-2} - CPI_{m-3})$,

with *d*: coupon date for which the *index number* is computed and *dd*: number of days in the month which the coupon date belongs to (rounding: 5 decimal places):

Coupon date	Coupon date – 2 months	Coupon date – 3 months	CPI m-2	CPI m-3	Index number
23/04/2014	28/02/2014	31/01/2014	100,0934	100,1867	100,1183
23/10/2014	31/08/2014	31/07/2014	100,3735	100,1867	100,3193
23/04/2015	28/02/2015	31/01/2015	99,7199	99,4398	99,6452
23/10/2015	31/08/2015	31/07/2015	100,2801	100,0934	100,2259
23/04/2016	29/02/2016	31/01/2016	99,5000	99,7000	99,5533
23/10/2016	31/08/2016	31/07/2016	100,2000	100,0000	100,1419
23/04/2017	28/02/2017	31/01/2017	101,0000	100,6000	100,8933
23/10/2017	31/08/2017	31/07/2017	101,4000	101,0000	101,2839
23/04/2018	28/02/2018	31/01/2018	101,5000	101,5000	101,5000
23/10/2018	31/08/2018	31/07/2018	102,0512	101,9800	102,0305
23/04/2019	28/02/2019	31/01/2019	102,4667	102,4024	102,4495
23/10/2019	31/08/2019	31/07/2019	103,0520	102,9478	103,0218
23/04/2020	29/02/2020	31/01/2020	103,6637	103,5662	103,6377

Once an *index number* for each coupon date has been computed it is possible to calculate the relative *indexation coefficient* (*IC*), bearing in mind that the *IC* at issue date is equal to 1 and that the following *IC*s are equal to:

$IC_t = \frac{\text{index\_number}_t}{\max(\text{index\_number}_{t-1}; \ldots; \text{index\_number}_0)}$ for *BTP Italia*,

that is the ratio of the *index number* relative to the coupon date for which the *IC* is computed and the maximum among the previous *index numbers* (rounding: 5 decimal places), and

$IC_t = \frac{\text{index\_number}_t}{\text{index\_number}_0}$ for other *linkers*,

that is the ratio of the *index number* relative to the coupon date for which the *IC* is computed and the issue date *index numbers* (rounding: 5 decimal places).

In the example below an example of computation of *ICs* for a *BTP Italia* is shown:

Coupon date	**Index number**	**IC**
23/04/2014	100,1183	1,0000
23/10/2014	100,3193	1,0020
23/04/2015	99,6452	0,9933
23/10/2015	100,2259	0,9991
23/04/2016	99,5533	0,9924
23/10/2016	100,1419	0,9982
23/04/2017	100,8933	1,0057
23/10/2017	101,2839	1,0039
23/04/2018	101,5000	1,0021
23/10/2018	102,0305	1,0052
23/04/2019	102,4495	1,0041
23/10/2019	103,0218	1,0056
23/04/2020	103,6377	1,0060

Since *BTP Italia*s guarantee real coupons, in case of deflation (*IC* < 1) a floor equal to 1 is applied to the *IC*s. The *adjusted IC*s can therefore be defined as *max (IC; 1)*:

Coupon date	**Index number**	**IC**	**Adjusted IC**
23/04/2014	100,1183	1,0000	1,0000
23/10/2014	100,3193	1,0020	1,0020
23/04/2015	99,6452	0,9933	1,0000
23/10/2015	100,2259	0,9991	1,0000
23/04/2016	99,5533	0,9924	1,0000
23/10/2016	100,1419	0,9982	1,0000
23/04/2017	100,8933	1,0057	1,0057
23/10/2017	101,2839	1,0039	1,0039
23/04/2018	101,5000	1,0021	1,0021
23/10/2018	102,0305	1,0052	1,0052
23/04/2019	102,4495	1,0041	1,0041
23/10/2019	103,0218	1,0056	1,0056
23/04/2020	103,6377	1,0060	1,0060

In case of other *linkers* it is only the last *IC* to be floored at 1.

It is then possible to compute each coupon the following way:

$$\text{coupon}_t = \frac{\text{real\_annual\_coupon\_rate}}{\text{coupon\_frequency}} * \text{principal} * IC_{\text{adjusted\_t}}:$$

Coupon date	Index number	IC	Adjusted IC	Coupon
23/04/2014	100,1183	1,0000	1,0000	-[2]
23/10/2014	100,3193	1,0020	1,0020	0,4133
23/04/2015	99,6452	0,9933	1,0000	0,4125
23/10/2015	100,2259	0,9991	1,0000	0,4125
23/04/2016	99,5533	0,9924	1,0000	0,4125
23/10/2016	100,1419	0,9982	1,0000	0,4125
23/04/2017	100,8933	1,0057	1,0057	0,4149
23/10/2017	101,2839	1,0039	1,0039	0,4141
23/04/2018	101,5000	1,0021	1,0021	0,4134
23/10/2018	102,0305	1,0052	1,0052	0,4147
23/04/2019	102,4495	1,0041	1,0041	0,4142
23/10/2019	103,0218	1,0056	1,0056	0,4148
23/04/2020	103,6377	1,0060	1,0060	0,4150

The revaluation of the principal amount is again differently treated:

BTP Italia:

The principal revaluation must be computed for each coupon date the following way:

$$\text{principal\_revaluation}_t = \text{principal} * \max(IC_t - 1; 0).$$

At maturity the principal reimbursement must be added to the final total payment.

Other *linkers:*

The revaluation of the principal amount is paid only at maturity. Therefore, before maturity the principal revaluation will be:

$$\text{principal\_revaluation}_{t<>T} = 0\ ,$$

while at maturity the principal revaluation will depend on the ratio between the last (*i.e.* at maturity date) and the first (*i.e.* at issue date) *index numbers*:

$$\text{principal\_revaluation}_{t=T} = \text{principal} * \max\left(\frac{\text{index\_number}_T}{\text{index\_number}_0}; 1\right).$$

The principal revaluation computed this way must be added to the previously computed coupon to get the final payment (rounding: 2 decimal places).

The example below again refers to a *BTP Italia*:

[2] Issue date

Coupon date	IC	Adjusted IC	Coupon	Principal revaluation	Payment
23/04/2014	1,0000	1,0000	-	-	-
23/10/2014	1,0020	1,0020	0,4133	0,2008	0,61
23/04/2015	0,9933	1,0000	0,4125	0,0000	0,41
23/10/2015	0,9991	1,0000	0,4125	0,0000	0,41
23/04/2016	0,9924	1,0000	0,4125	0,0000	0,41
23/10/2016	0,9982	1,0000	0,4125	0,0000	0,41
23/04/2017	1,0057	1,0057	0,4149	0,5722	0,99
23/10/2017	1,0039	1,0039	0,4141	0,3871	0,80
23/04/2018	1,0021	1,0021	0,4134	0,2134	0,63
23/10/2018	1,0052	1,0052	0,4147	0,5227	0,94
23/04/2019	1,0041	1,0041	0,4142	0,4107	0,82
23/10/2019	1,0056	1,0056	0,4148	0,5586	0,97
23/04/2020	1,0060	1,0060	0,4150	0,5978	101,01

Once the complete stream of payments has been computed, obviously only future payments are considered for *cash-flow mapping* purposes. The final table of future payments will thus be as follows (evaluation date: 20/04/2018):

Coupon date	IC	Adjusted IC	Coupon	Principal revaluation	Payment
23/04/2018	1,0021	1,0021	0,4134	0,2134	0,63
23/10/2018	1,0052	1,0052	0,4147	0,5227	0,94
23/04/2019	1,0041	1,0041	0,4142	0,4107	0,82
23/10/2019	1,0056	1,0056	0,4148	0,5586	0,97
23/04/2020	1,0060	1,0060	0,4150	0,5978	101,01

2.1.2 Bullet bonds

Bullet bonds (*bullets*) are the simplest category among those described in this paragraph: it is indeed sufficient to define the sequence of the future coupon dates and compute each coupon as:

$$(7)\ \text{coupon}_t = \text{principal\_amount} * \frac{\text{annual\_coupon\_rate}}{\text{coupon\_frequency}}.$$

For example, a *bullet* with a principal amount of 100 is assumed to pay semiannually a 5% annual rate every 30th of September and 31st of March until maturity (30th September 2020). If the evaluation date is 20/04/2018:

Date	Payment
30/09/2018	2,5
31/03/2019	2,5
30/09/2019	2,5
31/03/2020	2,5
30/09/2020	102,5

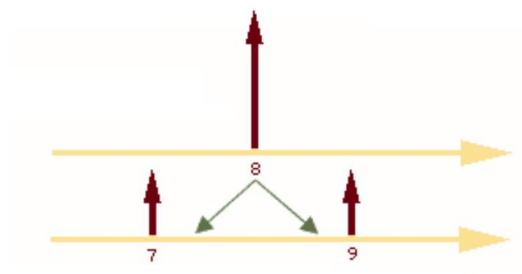
A particular sub-type of *bullets* are *zero-coupon bonds (ZCs)*, with 0% coupon rate.

2.2 Cash-flow mapping

Once the coupon stream for each of the securities in the portfolio subject to margining has been defined, cash flows can be mapped to their respective risk factors.

Since cash flows can be potentially infinite, a mapping system is used that allows to reduce their number and map them to a finite and relatively small set of *ZC curve* tenors called "vertices".

For example, assuming to have a single bond with a single cash flow in exactly 8 years and that 8 years is not a managed vertex of the reference *ZC curve*, this cash flow will be split into a pair of cash flows at year 7 and 9, if managed.



The two cash flows that originate from the original cash flow must be split in a way the current market value and the sign of the original cash flow are preserved.

This kind of mapping procedure, called *cash flow mapping*, allows to take into account the risk associated to each future cash flow generated by a bond, discounted at the appropriate rate, and also the non-perfect correlation between tenors (corresponding rates) of a *ZC curve* (as opposed to *duration mapping* or *principal mapping*).

In order to make the model manageable, cash flows, actually distributed along a continuum of maturities, are mapped to *n* tenors of the reference *ZC curve*. Employed *ZC curves* are Italian nominal, Italian real, Spanish nominal, Spanish real, Irish nominal and Portuguese nominal.

Cash flows with a maturity that does not coincide with one of the *n* maturities of the reference *ZC curve* are split on the two contiguous maturities, the one preceding and the one following the maturity of the cash flow, respecting the following three conditions:

- ***The market value must be preserved***: The market value of the two originated cash flows must be equal to the market value of the original cash flow.
- ***The market risk must be preserved***: The market risk of the two originated cash flows must be equal to the market risk of the original cash flow.
- ***The sign must be preserved***: The two originated cash flows must both have the same sign as the original cash flow.

In order to apply the *cash flow mapping* procedure it is necessary to:

1) compute the *time to payment (TTP)* of each cash flow of each bond;
2) compute the *yield to maturity* of the bond which the particular cash flows belong to and then compute the *market value* of these cash flows;
3) analyze the *ZC curve* on which each bond is mapped in terms of volatility of each tenor and correlation among tenors;
4) calculate the weights used to map each cash flow (*market value*) on the contiguous vertices of the reference *ZC curve*;
5) map each cash flow (*market value*) on the abovementioned vertices.

2.2.1 Time to payment definition

As previously mentioned, each security in the portfolio subject to margining is split into its future cash flows. For each of these cash flows it is necessary to identify the relative *time to payment*, as follows (act/act day count convention):

$$(8)\ \text{TTP} = \frac{\text{n days in period 365}}{365} + \frac{\text{n days in period 366}}{366}.$$

Formula (8) allows to take into account leap years, in case cash flows fall within them. In particular, the accrual periods within non-leap years and within leap years are identified for each cash flow:

Figure 2-2: *TTP* definition

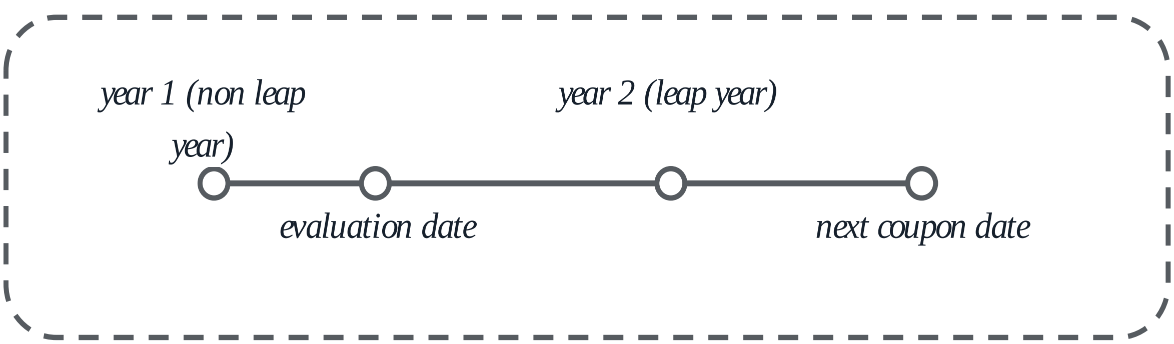


Figure 2-2 exemplifies a situation in which the time interval between 'evaluation date' and 'year 2' (start) constitutes the first term of (8), while the time interval between 'year 2' (start) and 'next coupon date' constitutes the second term of (8), with *n days in period 365 (366)*:

actual number of days between the two dates. If a cash flow accrues entirely in a non-leap/leap year the second/first term of (8) is set to 0.

Consider the following example:

Evaluation date: 20/04/2018;

Coupon date: 15/05/2020.

In this case the *time to payment* will be:

$$\text{TTP} = \frac{(31/12/2018 - 20/04/2018)}{365} + 1 + \frac{(15/05/2020 - 31/12/2019)}{366}.$$

It is then necessary to identify the issuer of each security in the portfolio subject to margining, in order to define on what *ZC curve* the security itself (its cash flows) will be mapped. We have to bear in mind that countries issuing both *nominal* and *inflation-linked* bonds will have two distinct *curves.*

For each security it is therefore necessary to build a table of the following type:

Table 10: Portfolio cash flow structure

Portfolio	ISIN	Issuer	TTP	Cash flow
X	IT000XXXXXXX	IT	TTP_1_bond_1	Cashflow_1_bond_1
X	IT000XXXXXXX	IT	TTP_2_bond_1	Cashflow_2_bond_1
X	ES00000XXXXX	ES	TTP_1_bond_2	Cashflow_1_bond_2
…	…	…	…	…

Each cash flow must therefore be assigned to the proper *ZC curve*, in order to map its amount on the respective contiguous vertices.

2.2.2 Yield to maturity and market value calculation

To each cash flow must be assigned the *yield to maturity* of the bond which it belongs to. The calculation of the *yield to maturity* of a bond is fundamental as it represents the *discount factor* which allows to compute the *market value* of each of its cash flows (*market value* which in turn will be mapped on the *ZC curve*).

In order to perform this calculation it is necessary to use a fitting algorithm (*e.g.* Newton) which, fed by the data, allows to obtain the *yield to maturity* of all the securities in the portfolio subject to margining. In particular, given the *dirty market price* of a security and the schedule of its cash flows, the relative *yield to maturity* can be computed according to the formula below:

$$\text{(9) bond\_price} = \left(\sum_{i=1}^{T-1} \left(\frac{\frac{\text{coupon}}{\text{coupon\_frequency}}}{(1+\text{YTM*})^{i}}\right)\right) + \frac{\frac{\text{coupon}}{\text{coupon\_frequency}} + \text{princpal}}{(1+\text{YTM*})^{T}},$$

with *i*: *time to payment* of the given coupon and *T*: *time to payment* of the cash flow at maturity. Given the guess value *YTM**, the chosen fitting algorithm will run until the difference between the *theoretical dirty price* (re)calculated according to the above formula and the *dirty market price* of the bond is below a predefined tolerance threshold.

Table 10 can then be integrated as follows:

Table 11: Portfolio cash flow structure and yield to maturity

Portfolio	ISIN	Issuer	TTP	Cash flow	Yield to maturity
X	IT000XXXXXXX	IT	TTP_1_bond_1	Cashflow_1_bond_1	Ytm_bond_1
X	IT000XXXXXXX	IT	TTP_2_bond_1	Cashflow_2_bond_1	Ytm_bond_1
X	ES00000XXXXX	ES	TTP_1_bond_2	Cashflow_1_bond_2	Ytm_bond_2
…	…	…	…	…	…

For each cash flow the relative *market value* must then be calculated by discounting the cash flow by the *yield to maturity* of the security it belongs to:

Table 12: Portfolio cash flow *market value*

TTP	Cash flow	Yield to maturity	Market value
TTP_1_bond_1	Cashflow_1_bond_1	Ytm_bond_1	Cashflow_1_bond_1 / (1 + Ytm_bond_1) ^ TTP_1_bond_1 * ps
TTP_2_bond_1	Cashflow_2_bond_1	Ytm_bond_1	Cashflow_2_bond_1 / (1 + Ytm_bond_1) ^ TTP_2_bond_1 * ps
TTP_1_bond_2	Cashflow_1_bond_2	Ytm_bond_2	Cashflow_1_bond_2 / (1 + Ytm_bond_2) ^ TTP_1_bond_2 * ps
…	…	…	…

with *ps*: *position sign* (i.e. +1 for long ISINs and -1 for short ISINs).

The sum of the *market values* of all the cash flows belonging to a given security has to be equal to the *market value* of the security itself.

2.2.3 Sovereign zero-coupon spot curve analysis

The time series of all the tenors (rates) of all the *ZC curves* must have length at least equal to the *lookback period* model parameter + 1:

Table 13: ZC curve tenors time series

Date	3M	…	30Y
t - n	w_t-n %	…	z_t-n %
t - n + 1	w_t-n+1 %	…	z_t-n+1 %
…	…	…	…
t	w_t %	…	z_t %

In case of '*all available data*' *lookback period* the time series will obviously be sufficient.

It is necessary to compute the following quantities:

1) *volatility* (σ) of each tenor of each *ZC curve*;
2) *correlation* (ρ) of each pair of contiguous tenors of each *ZC curve*.

In order to compute the *volatility* (σ) it is necessary to transform the rate time series into time series of daily rate absolute variations:

Table 14: *ZC curve* time series – daily variations

Date	3M	…	30Y
t - n			
t - n + 1	(w_t-n+1 - w_t-n) %	…	(z_t-n+1 - z_t-n) %
…	…	…	…
t	(w_t - w_t-1) %	…	(z_t - z_t-1) %

It is then possible to compute the (sample) *volatility* (σ) of each tenor according to formula:

$$(10)\ \sigma = \sqrt{\sum_{i=1}^{n} \frac{(x_i - x_{avg})^2}{n-1}},$$

with x_{avg}: average of the observations whose *volatility* is being computed (daily rate absolute variations).

Correlation (ρ) must be computed for all the pairs of contiguous tenors (of each *ZC curve*), with the exception of the last tenor, since there is no corresponding upper tenor.

For example, consider a *ZC curve* with the following structure:

3M	6M	1Y	2Y	3Y	4Y

Correlation (ρ) must be computed for the pairs of tenors 3M/6M, 6M/1Y, 1Y/2Y, 2Y/3Y and 3Y/4Y. The computation of ρ must be performed according to the following formula:

$$(11)\ \rho = \left(\sum_{i=1}^{n} (x_i - x_{avg})(y_i - y_{avg})/(n-1)\right) \Big/ \sqrt{\sum_{i=1}^{n} \frac{(x_i - x_{avg})^2}{n-1} \sum_{i=1}^{n} \frac{(y_i - y_{avg})^2}{n-1}},$$

with *x* and *y*: *down* and *up* tenors, respectively.

For example, consider the following time series of daily rate absolute variations for the 3-month and 6-month tenors of a specific *ZC curve*:

Date	3M	6M
12/04/2018	0,725%	0,725%
13/04/2018	0,543%	0,543%
16/04/2018	0,543%	0,283%
17/04/2018	0,972%	0,972%
18/04/2018	0,445%	0,445%
19/04/2018	0,445%	0,445%
20/04/2018	1,656%	1,656%

The *lookback period* in the example is equal to 7. x_{avg} and y_{avg} are equal to 0,761% e 0,724%, respectively. According to formulas (10) and (11), the 3 month- and 6 month-tenor *volatilities* are equal to 0,436% and 0,468%, respectively. Their *correlation* is instead equal to 97,88%.

2.2.4 Weight calculation for cash-flow mapping

Once the *TTP* and the *market value* of each cash flow of each bond in the portfolio subject to margining have been computed, it is then possible to map each of these cash flows (their *market values*) on the tenors of the reference *ZC curves* (i.e. the *curve* of the country issuing the bond).

For example, consider the following margining portfolio:

Portfolio	ISIN	Issuer	TTP	Cash flow market value
X	IT000XXXXXX1	IT	0,3	100.000
X	IT000XXXXXX2	IT	1,2	150.000

and Italian *ZC curve* structure:

3M	6M	1Y	2Y

Expressing the *TTP* in year fractions, the first cash flow (*TTP*: 0,3) has to be mapped between onto the 3 month- (0,25) and 6 month- (0,5) tenors. The second cash flow has instead to be mapped onto the 1 year- (1) and 2 year- (2) tenors:[3]

ISIN	Issuer	TTP	Market value	Down tenor	Up tenor
IT000XXXXXX1	IT	0,3	100.000	0,25	0,5
IT000XXXXXX2	IT	1,2	150.000	1	2

Once the relevant tenors of the reference *ZC curve* have been identified, the *down* and *up* weights of each cash flow must be computed: these weights allow to map a certain fraction of the cash flow on the pair of relevant tenors within which the cash flow falls, in compliance with the principles outlined at the beginning of this paragraph.

It is therefore necessary to compute the *interpolation coefficients* φ_{down} and φ_{up} for each of the cash flows to map: these are function of the *TTP* of the cash flow and of the *duration* of the *down* and *up* tenors:

(12) $\varphi_{up} = \frac{\text{TTP - down\_tenor}}{\text{up\_tenor - down\_tenor}}$;

(13) $\varphi_{down} = (1 - \varphi_{up})$,

[3] In case a cash flow has a *TTP* lower (higher) than the shortest (longest) tenor, it has to be entirely mapped on the latter.

thus obtaining:

TTP	Market value	Down tenor	Up tenor	φ_{low}	φ_{up}
0,3	100.000	0,25	0,5	0,8	0,2
1,2	150.000	1	2	0,8	0,2

The *volatilities* of the *down* (σ_n) and *up* (σ_{n+1}) tenors of a cash flow, computed according to formula (10), are then multiplied by the respective *interpolation coefficients* (the *volatility* of the *down* tenor must be multiplied by the *interpolation coefficient* assigned to the *down* tenor φ_{down}, the *volatility* of the *up* tenor by the *interpolation coefficient* of the *up* tenor φ_{up}), this way obtaining σ^*_n and σ^*_{n+1}, i.e. the *volatilities* of the *down* and *up* tenors of the cash flow *adjusted* by the respective *interpolation coefficient.*

By employing σ^* instead of σ it is possible to prevent the distortions that would arise in case the *volatilities* of two contiguous tenors were not regularly one greater than the other (with consequent abrupt fluctuations in the weights applied to the cash flows involved and hence fluctuations in the margin requirements not justified by changes in the riskiness of the portfolio itself).

The use of σ^* in the mapping procedure also allows to map the cash flows consistently with their positioning within the interval represented by the two contiguous vertices.

Let's indicate with W_n and W_{n+1} the weights of the *down* and *up* tenors, respectively, with σ^*_n and σ^*_{n+1} their *adjusted volatilities* and with $\rho_{n,n+1}$ their *correlation*, according to formula (11).

Since the sum of the two weights must be equal to 1 (in order to respect the principle according to which the *market value* of the original cash flow to be mapped must be preserved), we have that:

$$W_{n+1} = 1 - W_n .$$

Furthermore,

$$\sigma^*_{int} = \sqrt{W_n^2 \sigma^{*2}_n + W_{n+1}^2 \sigma^{*2}_{n+1} + 2 W_n W_{n+1} \sigma^*_n \, \sigma^*_{n+1} \rho_{n,n+1}} .$$

We can therefore compute

$$(14)\quad W_n = \frac{-(2\sigma^*_n \, \sigma^*_{n+1}\rho_{n,n+1} - 2\sigma^{*2}_{n+1}) \pm \sqrt{\left(2\sigma^*_n \, \sigma^*_{n+1}\rho_{n,n+1} - 2\sigma^{*2}_{n+1}\right)^2 - 4(\sigma^{*2}_n + \sigma^{*2}_{n+1} - 2\sigma^*_n \, \sigma^*_{n+1}\rho_{n,n+1})(\sigma^{*2}_{n+1} - \sigma^{*2}_{int})}}{2(\sigma^{*2}_n + \sigma^{*2}_{n+1} - 2\sigma^*_n \, \sigma^*_{n+1}\rho_{n,n+1})},$$

with $\sigma^*_{int} = \varphi_{down} \, \sigma^*_n + \varphi_{up} \, \sigma^*_{n+1}$.

The fundamental theorem of algebra implies that formula (14) yields two solutions: in order to respect the principle according to which the *sign* of the original cash flow to be mapped must be preserved, it is necessary to choose the value of W_n which is between 0 and 1.

Based on the above, each cash flow can then be mapped on the *down* and *up* tenors, after having multiplied its *market value* by the respective weights:

TTP	Market vale	W_n	W_{n+1}	Cash flow down	Cash flow up
TTP_1	Marketvalue_cashflow_1	w	x	Marketvalue_cashflow_1 * w	Marketvalue_cashflow_1 * x
TTP_2	Marketvalue_cashflow_2	y	z	Marketvalue_cashflow_2 * y	Marketvalue_cashflow_2 * z

To summarize, for each ISIN in the portfolio subject to margining the structure of its future cash flows is defined. The relative *market value* is then computed (with positive or negative sign depending on the nature of the position) and mapped on the contiguous tenors of the reference *ZC curve*, according to the value of the statistical quantities characterizing the *curve* tenors themselves and to the *TTP* of the cash flow.

2.2.5 Market value mapping on sovereign zero-coupon spot curve tenors

For each ISIN in the portfolio subject to margining it is then possible to obtain, by adding up all the *market values* mapped on a specific *ZC curve* tenor, a structure like that represented below:

Table 15: Cash-flow mapping per ISIN

Portfolio	ISIN	Position	Tenor	Mapped cash flow (market value)
X	IT000XXXXXX1	L	Tenor_1	Sum_mapped_cashflows_bond_1
X	IT000XXXXXX1	L	Tenor_2	Sum_mapped_cashflows_bond_1
…	…	…	…	…
Portfolio	**ISIN**	**Position**	**Tenor**	**Mapped cash flow (market value)**
X	IT000XXXXXX2	S	Tenor_1	Sum_mapped_cashflows_bond_2
X	IT000XXXXXX2	S	Tenor_2	Sum_mapped_cashflows_bond_2
…	…	…	…	…

For each sovereign among those subject to cash-flow mapping (i.e. Italy – nominal/real subsets, Spain – nominal/real subsets, Portugal and Ireland) it is possible to compute the sum of the *market values* mapped on each tenor of the relative *ZC curve* (with netting of potential long and short values mapped on the same tenor), by adding up all the *market values* of the ISINs constituting the specific sovereign sub-portfolio:

Table 16: Cash-flow mapping per ZC curve

Portfolio	Issuer	Tenor	Total mapped cash flows (market value)
X	Y	Tenor_1	Sum_mapped_cashflows_issuer_Y
X	Y	Tenor_2	Sum_mapped_cashflows_issuer_Y
…	…	…	…

3 Price scenarios

3.1 Scaling of the sovereign zero-coupon spot curve time series

The *ZC curves* to which the *scaling* process is applied are all those employed to map the (market value of the) cash flows of the margined portfolio, i.e. relating to the issuers of the bonds in it. These are:

- Italian nominal;
- Italian real;
- Spanish nominal;
- Spanish real;
- Irish nominal;
- Portuguese nominal.

All these *curves* will be taken starting from mid-2004 for complete availability reasons.

The time series of each tenor of each reference *ZC curve* will have length equal to the *lookback period* (model parameter) plus:

- the *scaling window* (model parameter) employed in the calculation of the *EWMA volatilities*;
- the *holding period* (model parameter) employed in the calculation of the *relative price returns*.

In case of '*all available data*' *lookback period* this will obviously be equal to *all available data – scaling window – holding period*.

If we call *n* the *lookback period* and *t* the *scaling window*, the panel data of a given *ZC curve* (assuming that the longest tenor has a duration of 30 years) can be generalized as follows:

Table 17: *ZC curve* panel data

n	3M	6M	1Y	…	30Y
1	x_1 %	y_1 %	v_1 %	…	w_1 %
…	…	…	…	…	…
…	…	…	…	…	…
…	…	…	…	…	…
n + t + hp - 1	x_n+t+hp-1 %	y_n+t+hp-1 %	v_n+t+hp-1 %	…	w_n+t+hp-1 %
n + t + hp	x_n+t+hp %	y_n+t+hp %	v_n+t+hp %	…	w_n+t+hp %

3.1.1 Conversion of the times series from rates to prices

The above *rate* time series must be converted into *price* time series employing the following formulas:

ZC curve tenors with duration < 1 year:

(1) $\text{price}= \frac{100}{(1+\text{rate})^d}$;

ZC curve tenors with duration >= 1 year:

(2) $\text{price}= 100*e^{-\text{rate}*d}$,

with *rate*: *rate%* / 100 and *d*: duration (in years) of the tenor of the *ZC curve* whose *rate* time series is being converted into *price* time series.

Table 18: *Rates* into *prices* conversion

n	3M	6M	1Y	…	30Y
1	100 / (1 + x_1) ^ 0,25	100 / (1 + y_1) ^ 0,5	100 * exp(-1 * v_1)	…	100 * exp(-30 * w_1)
…	…	…	…	…	…
…	…	…	…	…	…
…	…	…	…	…	…
n + t + hp - 1	100 / (1 + x_n+t+hp-1) ^ 0,25	100 / (1 + y_n+t+hp-1) ^ 0,5	100 * exp(-1 * v_n+t+hp-1)	…	100 * exp(-30 * w_n+t+hp-1)
n + t + hp	100 / (1 + x_n+t+hp) ^ 0,25	100 / (1 + y_n+t+hp) ^ 0,5	100 * exp(-1 * v_n+t+hp)	…	100 * exp(-30 * w_n+t+hp)

3.1.2 Computation of the (unscaled) relative price returns

Once the *price* time series of each tenor of each *ZC curve* has been computed, it is necessary to compute the (*unscaled*) *relative price return* time series as follows:

(3) $price\_return_t = \frac{price_t}{price_{t-hp}} - 1$.

The time series computed this way has length equal to $n + t$.

For example, consider the following *rate* time series:

Date	1Y rate
14/03/2017	-0,149%
15/03/2017	-0,167%
16/03/2017	-0,184%
17/03/2017	-0,174%
20/03/2017	-0,172%
21/03/2017	-0,178%
22/03/2017	-0,176%
23/03/2017	-0,175%
24/03/2017	-0,180%
27/03/2017	-0,178%
28/03/2017	-0,175%

29/03/2017	-0,183%
30/03/2017	-0,176%
31/03/2017	-0,180%
03/04/2017	-0,189%

The above *rate* time series is converted into a *price* time series employing formula (2):

Date	1Y rate	1Y price
14/03/2017	-0,149%	100,150
15/03/2017	-0,167%	100,167
16/03/2017	-0,184%	100,184
17/03/2017	-0,174%	100,174
20/03/2017	-0,172%	100,172
21/03/2017	-0,178%	100,178
22/03/2017	-0,176%	100,176
23/03/2017	-0,175%	100,175
24/03/2017	-0,180%	100,181
27/03/2017	-0,178%	100,178
28/03/2017	-0,175%	100,175
29/03/2017	-0,183%	100,183
30/03/2017	-0,176%	100,176
31/03/2017	-0,180%	100,181
03/04/2017	-0,189%	100,189

Assuming a 5 day-*holding period* the *relative price return* time series can be represented as follows:

Date	1Y rate	1Y price	1Y relative price return
14/03/2017	-0,149%	100,150	
15/03/2017	-0,167%	100,167	
16/03/2017	-0,184%	100,184	
17/03/2017	-0,174%	100,174	
20/03/2017	-0,172%	100,172	
21/03/2017	-0,178%	100,178	0,029%
22/03/2017	-0,176%	100,176	0,009%
23/03/2017	-0,175%	100,175	-0,009%
24/03/2017	-0,180%	100,181	0,007%
27/03/2017	-0,178%	100,178	0,006%
28/03/2017	-0,175%	100,175	-0,004%
29/03/2017	-0,183%	100,183	0,007%
30/03/2017	-0,176%	100,176	0,001%
31/03/2017	-0,180%	100,181	0,000%
03/04/2017	-0,189%	100,189	0,011%

3.1.3 Computation of the EWMA volatility

Given the *relative price return* time series computed as described above, it is then necessary to compute for each observation of the *lookback period* the corresponding value of the *volatility* according to the *EWMA* methodology.

In particular, a *seed volatility* is computed on the first *scaling window t* observations of the time series according to formula (10) of above

Cash-flow mapping section.

For the following *lookback period* *n* observations of the time series the volatility of each observation is recursively computed according to the formula:

$$(4)\quad \sigma_i = \sqrt{\lambda\sigma_{i-1}^2 + (1-\lambda)r_i^2}$$

$$\sigma_1 = \sqrt{\lambda\sigma_0^2 + (1-\lambda)r_1^2}$$

$$\sigma_2 = \sqrt{\lambda\sigma_1^2 + (1-\lambda)r_2^2} = \sqrt{\lambda(\lambda\sigma_0^2 + (1-\lambda)r_1^2) + (1-\lambda)r_2^2}\,,$$

with λ: *smoothing factor* (comprised between 0 and 1); *r*: *relative price return* computed according to formula (3). Formula (4), which is a variant of the formula $\sigma_i = \sqrt{\lambda\sigma_{i-1}^2 + (1-\lambda)r_{i-1}^2}$, allows, as outlined at the beginning of the document, to weigh the observations based on the current volatility cluster.

For example, consider a 11 day-*scaling window*, a 8 day-*lookback period* (evaluation date: 15/04/2017, assuming last available *ZC curve* data is 14/04/2017) and a *smoothing factor* λ = 0,94. Furthermore, consider the following *relative price return* time series:

Date	Relative price return	EWMA volatility	Notes
21/03/2017	0,029%		
22/03/2017	0,009%		*SCALING WINDOW*
23/03/2017	-0,009%		
24/03/2017	0,007%		
27/03/2017	0,006%		
28/03/2017	-0,004%		
29/03/2017	0,007%		* standard deviation of the observations between 21/03/2017 and 04/04/2017
30/03/2017	0,001%		
31/03/2017	0,000%		
03/04/2017	0,011%		
04/04/2017	0,019%	**0,010%***	
05/04/2017	0,010%	0,010%	
06/04/2017	0,024%	0,011%	
07/04/2017	0,027%	0,013%	*LOOKBACK PERIOD*
10/04/2017	0,005%	0,013%	
11/04/2017	-0,014%	0,013%	
12/04/2017	-0,021%	0,013%	Formula (4) is applied
13/04/2017	-0,029%	0,015%	
14/04/2017	-0,034%	0,017%	

3.1.4 Scaled relative price returns

Given the *relative price return* time series[4] and the *volatility* computed according to the *EWMA* methodology outlined in the previous paragraph, it is then possible to compute the *scaled relative price return* time series.

In particular, the *scaling factor* applied to each observation is computed according to the following formula (mid-volatility approach):

(5) $scaling\_factor_t = \frac{\sigma_T+\sigma_t}{2*\sigma_t}$,

with σ_T: *EWMA volatility* computed for the most recent observation of the time series (therefore, evaluation day - 1 day); σ_t: *EWMA volatility* computed for the specific observation which the *scaling factor* is applied to (full-volatility approach would have been characterized by the formula $scaling\_factor_t = \frac{\sigma_T}{\sigma_t}$).

The *scaling factor* is applied to each observation of the *relative price return* time series according to the following formula:

(6) $scaled\_price\_return_t = unscaled\_price\_return_t * scaling\_factor_t$

Consider the previous example time series (evaluation date: 15/04/2017): σ_T is equal to 0,017% and the *scaled relative price return* time series is as follows:

Date	Unscaled relative price return	EWMA volatility	Scaled relative price return
05/04/2017	0,010%	0,010%	0,014%
06/04/2017	0,024%	0,011%	0,031%
07/04/2017	0,027%	0,013%	0,031%
10/04/2017	0,005%	0,013%	0,006%
11/04/2017	-0,014%	0,013%	-0,016%
12/04/2017	-0,021%	0,013%	-0,024%
13/04/2017	-0,029%	0,015%	-0,031%
14/04/2017	-0,034%	0,017%	-0,034%

3.2 Price scenarios definition

It is then necessary to define a series of *price scenarios* with length equal to the *lookback period*. Each *price scenario* is computed on the basis of the chosen *holding period* as the ratio between the observation for which the specific *price scenario* is being calculated and the *hp* day-earlier observation (e.g. if *hp* is equal to 5 days each *price scenario* is computed as the ratio between the current observation and the 5-day earlier observation):

[4] The part of the time series of interest is that post-*scaling window*. Its length is therefore equal to *n* (chosen *lookback period*).

(7) $price\_scenario_t = \frac{price_t}{price_{t-hp}}$.

Both *scaled* and *unscaled price scenarios* are computed.

3.2.1 Scaled price scenarios

In particular, the *scaled price scenarios* can be computed employing the *scaled relative price return* time series. Each *relative price return* is indeed already computed as $relative\_price\_return_t = \frac{price_t}{price_{t-hp}} - 1$. One can thus compute the *price scenario* as:

(8) $scaled\_price\_scenario_t = scaled\_relative\_price\_return_t + 1$.

It is possible to summarize the process for obtaining the *scaled price scenarios* the following way:

Table 19: Scaled price scenarios computation

Date	Unscaled relative price return	Scaled relative price return	Scaled price scenario
1	price_1 / price_1-hp - 1	(price_1 / price_1-hp – 1) * scaling_factor_1	(price_1 / price_1-hp – 1) * scaling_factor_1 + 1
…	…	…	…
…	…	…	…
n	price_n / price_n-hp - 1	(price_n / price_n-hp – 1) * scaling_factor_n	(price_n / price_n-hp – 1) * scaling_factor_n + 1

The methodology for calculating the *scaling factor* is described in the previous section.

Always considering the previous example, we therefore have:

Date	Scaled relative price return	Scaled price scenario
05/04/2017	0,014%	1,00014
06/04/2017	0,031%	1,00031
07/04/2017	0,031%	1,00031
10/04/2017	0,006%	1,00006
11/04/2017	-0,016%	0,99984
12/04/2017	-0,024%	0,99976
13/04/2017	-0,031%	0,99969
14/04/2017	-0,034%	0,99966

3.2.2 Unscaled price scenarios

The *unscaled price scenarios* can instead be computed simply skipping, with reference to what outlined in the previous paragraph, the *relative price return-scaling factor* multiplication step. It is indeed sufficient to compute the *price scenarios* employing formula (7):

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Table 20: Unscaled price scenarios computation

Date	Unscaled price scenario
1	price_1 / price_1-hp
…	…
…	…
n	price_n / price_n-hp

4 Expected Shortfall

The *Expected Shortfall* (*ES*) is a risk measure consisting in the average of the tail events of a given distribution. It is preferred to the *Value at Risk* (*VaR*) risk measure, which basically consists of the quantile of that distribution above which the tail actually 'starts', as it is coherent and more conservative. It is also called *Conditional-VaR* (*C-VaR*).

The risk measure can either be *undiversified* or *diversified*, depending on whether it is computed 'per block' (*i.e.* without inter-country diversification benefits) or 'as a unique block' (*i.e.* allowing those benefits). Practically speaking, in one case (the *undiversified* case) the country portfolios are revalued separately from one another in their set of historical scenarios and the respective risk measures are computed; in the other case (the *diversified* case) the portfolio is revalued as a whole and a unique risk measure is computed (therefore there will be a unique set of historical scenarios).

Whatever the particular choice is, the current market value of a portfolio is revalued in a set of historical scenarios. These revaluations are then compared to the former and a set of profits/losses is obtained. This P&L distribution will be characterized by some extreme profits in one tail and some extreme losses in the other tail.

4.1 Undiversified Expected Shortfall calculation

4.1.1 Undiversified Expected Shortfall (per country)

In order to compute the *undiversified Expected Shortfall* consider a simple hypothetical portfolio consisting of bonds issued by a single country, whose cash flows (at market value) are mapped on the first 3 tenors only of the reference *ZC curve*. The cash-flow mapping structure can be represented as follows:

Table 21: Margined portfolio cash-flow mapping

Tenor	Cash flows mapped
3M	Cashflow_3M
6M	Cashflow_6M
1Y	Cashflow_1Y

Consider also the following *n* (chosen *lookback period*) *scaled / unscaled price scenarios* defined according to the methodology outlined in the previous section:

Table 22: Price scenarios

Date	3M	6M	1Y
1	Pricescenario_1_3M	Pricescenario_1_6M	Pricescenario_1_1Y
2	Pricescenario_2_3M	Pricescenario_2_6M	Pricescenario_2_1Y
…	…	…	…
n-1	Pricescenario_n-1_3M	Pricescenario_n-1_6M	Pricescenario_n-1_1Y

n	Pricescenario_n_3M	Pricescenario_n_6M	Pricescenario_n_1Y

The market value of the cash flows mapped on each relevant tenor of the *ZC curve* must be revalued in each *price scenario*:

Table 23: Cash flows revaluation per tenor

Date	3M	6M	1Y
1	Pricescenario_1_3M * Cashflow_3M	Pricescenario_1_6M * Cashflow_6M	Pricescenario_1_1Y * Cashflow_1Y
2	Pricescenario_2_3M * Cashflow_3M	Pricescenario_2_6M * Cashflow_6M	Pricescenario_2_1Y * Cashflow_1Y
…	…	…	…
n-1	Pricescenario_n-1_3M * Cashflow_3M	Pricescenario_n-1_6M * Cashflow_6M	Pricescenario_n-1_1Y * Cashflow_1Y
n	Pricescenario_n_3M * Cashflow_3M	Pricescenario_n_6M * Cashflow_6M	Pricescenario_n_1Y * Cashflow_1Y

Once the revalued (per tenor and *price scenario*) market value of each cash flow has been computed, it is possible to compute the revalued market value of the entire portfolio subject to margining in each *price scenario*:

Table 24: Portfolio revaluation

Date	Revalued portfolio
1	Pricescenario_1_3M * Cashflow_3M + Pricescenario_1_6M * Cashflow_6M + Pricescenario_1_1Y * Cashflow_1Y
2	Pricescenario_2_3M * Cashflow_3M + Pricescenario_2_6M * Cashflow_6M + Pricescenario_2_1Y * Cashflow_1Y
…	…
n-1	Pricescenario_n-1_3M * Cashflow_3M + Pricescenario_n-1_6M * Cashflow_6M + Pricescenario_n-1_1Y * Cashflow_1Y
n	Pricescenario_n_3M * Cashflow_3M + Pricescenario_n_6M * Cashflow_6M + Pricescenario_n_1Y * Cashflow_1Y

Having the revalued market value of the portfolio subject to margining in each *price scenario* and its current market value it is possible to compute its profit/loss in each *price scenario*:

Table 25: Portfolio profit/loss per price scenario

Date	Revalued portfolio	Current portfolio	Profit/Loss
1	Pricescenario_1_3M * Cashflow_3M + Pricescenario_1_6M * Cashflow_6M + Pricescenario_1_1Y * Cashflow_1Y = *Revalued market value 1*	*Current market value*	*Revalued market value 1 – Current market value*
2	Pricescenario_2_3M * Cashflow_3M + Pricescenario_2_6M * Cashflow_6M + Pricescenario_2_1Y * Cashflow_1Y = *Revalued market value 2*	*Current market value*	*Revalued market value 2 – Current market value*
…	…	…	…
n-1	Pricescenario_n-1_3M * Cashflow_3M + Pricescenario_n-1_6M * Cashflow_6M + Pricescenario_n-1_1Y * Cashflow_1Y = *Revalued market value n-1*	*Current market value*	*Revalued market value n-1 – Current market value*
n	Pricescenario_n_3M * Cashflow_3M + Pricescenario_n_6M * Cashflow_6M + Pricescenario_n_1Y * Cashflow_1Y = *Revalued market value n*	*Current market value*	*Revalued market value n – Current market value*

Having the portfolio profit/loss in each *price scenario* it is possible to compute the portfolio *undiversified Expected Shortfall* according to two different approaches:

- *Single tail* approach *(worst losses):*

The *single tail* approach implies that only losses are considered. These losses are sorted from the worst to the less serious and, given the chosen *confidence level,* the portfolio *undiversified Expected Shortfall* is computed as average of the tail observations.

For example, consider a 5 day-*lookback period*, a 80% *confidence level* and the following set of portfolio profits/losses (net long position):

Date	Revalued portfolio	Current portfolio	Profit/Loss
1	10	10	0
2	8	10	-2
3	12	10	2
4	7	10	-3
5	7,5	10	-2,5

We sort the profits/losses from the worst loss to best profit and obtain:

Date	Revalued portfolio	Current portfolio	Gain/Loss
4	7	10	-3
5	7,5	10	-2,5
2	8	10	-2
1	10	10	0
3	12	10	2

It is then necessary to compute the number of observations in the tail of the P&L distribution given the chosen *lookback period* and *confidence level*, as *number of observations*[5] ** (1 – confidence level)*, rounding the result to the nearest integer. In the example the number of tail observations is then equal to 1. The *undiversified Expected Shortfall* of the portfolio is equal to the average of the tail observations (in absolute terms). In the example it amounts to 3.

- *Double tail* approach *(worst absolute variations):*

The *double tail* approach implies that all variations are considered, in absolute terms. These absolute variations are sorted from the greatest to the smallest and, given the chosen *confidence level*, the portfolio *undiversified Expected Shortfall* is computed as average of tail observations.

For example, consider a 5 day-*lookback period*, a 80% *confidence level* and the following set of portfolio absolute variations:

Date	Revalued portfolio	Current portfolio	Profit/Loss absolute value
1	10	10	0
2	8	10	2
3	12	10	2
4	7	10	3
5	7,5	10	2,5

We sort the absolute variations from the greatest to the smallest and obtain:

Date	Revalued portfolio	Current portfolio	Profit/Loss absolute value
4	7	10	3
5	7,5	10	2,5
2	8	10	2
3	12	10	2
1	10	10	0

It is then necessary to calculate the number of tail observations given the chosen *lookback period* and *confidence level*, as *number of observations*[6] ** (1 – confidence level)*, rounding the result to the nearest integer. In the example the number of tail observations is then equal to 1. The

[5] Equal to the *lookback period*.
[6] Equal to the *lookback period*.

undiversified Expected Shortfall of the portfolio is equal to the average of the selected observations, in the example equal to 3.

In case the portfolio contains bonds issued by different sovereigns, the methodology described above must be replicated for each sub-portfolio (consisting of all and only the ISINs issued by a specific country and therefore mapped on a specific *ZC curve*). The portfolio *undiversified Expected Shortfall* is then equal to the sum of the *undiversified Expected Shortfalls* of each sub-portfolio.

It is worthwhile mentioning that for countries such as Italy and Spain having both nominal and real *ZC curves* the adopted country-approach would be the *diversified* one. Therefore, Italian nominal and real sub-portfolios would lead to a unique *diversified* country *Expected Shortfall*; the same can be said about Spain; finally, all country *Expected Shortfalls* would be summed up in an *undiversified* way.

4.1.2 Undiversified Expected Shortfall per sovereign zero-coupon bond tenor

Looking at the *undiversified Expected Shortfall* from a different and narrower (than per country) point of view, it is possible to compute the *undiversified Expected Shortfall* per *ZC curve* tenor.

Going back to Table 23, instead of proceeding as described further, the revalued market value per tenor-*price scenario* combination is directly compared to the market value of the sub-portfolio mapped on that specific tenor. This means that an *undiversified Expected Shortfall* for each tenor of the *ZC curves* involved in the cash-flow mapping can be computed.

For example, consider the 3 month tenor in Table 23:

Date	3M
1	Pricescenario_1_3M * Cashflow_3M
2	Pricescenario_2_3M * Cashflow_3M
…	…
n-1	Pricescenario_n-1_3M * Cashflow_3M
n	Pricescenario_n_3M * Cashflow_3M

The P&L distribution for that tenor can be computed as follows:

Date	Revalued tenor	Current tenor	Profit/Loss
1	Pricescenario_1_3M * Cashflow_3M = *Revalued market value 1*	Cashflow_3M	*Revalued market value 1* – Cashflow_3M
2	Pricescenario_2_3M * Cashflow_3M = *Revalued market value 2*	Cashflow_3M	*Revalued market value 2* – Cashflow_3M
…	…	…	…
n-1	Pricescenario_n-1_3M * Cashflow_3M =	Cashflow_3M	*Revalued market value n-1* – Cashflow_3M

	Revalued market value n-1		
n	Pricescenario_n_3M * Cashflow_3M = *Revalued market value n*	Cashflow_3M	*Revalued market value n* – Cashflow_3M

The calculation of the *undiversified Expected Shortfall* per single tenor follows the same logic (including the *single tail* / *double tail* approach distinction) as that described in the previous paragraph.

4.1.3 Diversified Expected Shortfall calculation per country

As opposed to the *undiversified Expected Shortfall* calculation outlined above, the calculation of the *diversified Expected Shortfall* is characterized by the acknowledgement of the investment diversification benefit to the Clearing Member (of course only in case its portfolio contains bonds issued by more than one country).

For example, consider the following cash flow structure:

Table 26: Margining portfolio cash-flow mapping (2)

Tenor	Cash flows mapped
3M_ITA	Cashflow_3M_ITA
6M_SPA	Cashflow_6M_SPA

Consider also the following *n* (equal to the chosen *lookback period*) *scaled* / *unscaled price scenarios* defined according to the methodology outlined in previous section:

Table 27: Price scenarios (2)

Date	3M_ITA	6M_SPA
1	Pricescenario_1_3M_ITA	Pricescenario_1_6M_SPA
2	Pricescenario_2_3M_ITA	Pricescenario_2_6M_SPA
…	…	…
n-1	Pricescenario_n-1_3M_ITA	Pricescenario_n-1_6M_SPA
n	Pricescenario_n_3M_ITA	Pricescenario_n_6M_SPA

The cash flows mapped on each relevant tenor of each reference *ZC curve* are revalued in each *price scenario*:

Table 28: Cash flows revaluation per tenor (2)

Date	Revalued ITA sub-portfolio	Revalued SPA sub-portfolio
1	Pricescenario_1_3M_ITA * Cashflow_3M_ITA	Pricescenario_1_6M_SPA * Cashflow_6M_SPA
2	Pricescenario_2_3M_ITA * Cashflow_3M_ITA	Pricescenario_2_6M_SPA * Cashflow_6M_SPA
…	…	…
n-1	Pricescenario_n-1_3M_ITA * Cashflow_3M_ITA	Pricescenario_n-1_6M_SPA * Cashflow_6M_SPA
n	Pricescenario_n_3M_ITA *	Pricescenario_n_6M_SPA *

	Cashflow_3M_ITA	Cashflow_6M_SPA

Since in this example each sub-portfolio consists of cash flows mapped on a single tenor of a *ZC curve*, Table 28 already represents the revalued country sub-portfolio. If there were cash flows mapped on more than one tenor per *ZC curve*, for each sub-portfolio it would have been necessary to make a calculation similar to that shown in Table 24.

The revalued market value of the entire portfolio in each *price scenario* is computed the following way:

Table 29: Cash flows revaluation per tenor

Date	Revalued ITA sub-portfolio	Revalued SPA sub-portfolio	Revalued portfolio
1	Pricescenario_1_3M_ITA * Cashflow_3M_ITA	Pricescenario_1_6M_SPA * Cashflow_6M_SPA	Pricescenario_1_3M_ITA * Cashflow_3M_ITA + Pricescenario_1_6M_SPA * Cashflow_6M_SPA
2	Pricescenario_2_3M_ITA * Cashflow_3M_ITA	Pricescenario_2_6M_SPA * Cashflow_6M_SPA	Pricescenario_2_3M_ITA * Cashflow_3M_ITA + Pricescenario_2_6M_SPA * Cashflow_6M_SPA
…	…	…	
n-1	Pricescenario_n-1_3M_ITA * Cashflow_3M_ITA	Pricescenario_n-1_6M_SPA * Cashflow_6M_SPA	Pricescenario_n-1_3M_ITA * Cashflow_3M_ITA + Pricescenario_n-1_6M_SPA * Cashflow_6M_SPA
n	Pricescenario_n_3M_ITA * Cashflow_3M_ITA	Pricescenario_n_6M_SPA * Cashflow_6M_SPA	Pricescenario_n_3M_ITA * Cashflow_3M_ITA + Pricescenario_n_6M_SPA * Cashflow_6M_SPA

The way to compute the *diversified Expected Shortfall* of the portfolio is the same as that described above (the distinction between *single tail* and *double tail* approaches still applying).

4.1.4 Use of Spectral Risk Measures (SRMs)

We anticipated the *Expected Shortfall* is the average of a set of tail events. The 'plain' *Expected Shortfall* is indeed a simple average, *i.e.* each of the *n* tail events has a weight of $1/n$. However, it is also possible using non-uniform weighting schemes.

In particular, long *lookback periods* may imply very diluted tails. When the *all available data lookback period* is chosen, given a fixed *confidence level*, the number of observations in the tails is destined to increase every passing day. Since the *Expected Shortfall* represents the average of these observations, a tail which keeps diluting implies an *Expected Shortfall* risk measure less and less conservative. A trade-off therefore exists between longer *lookback periods* (allowing to always take into account past significant stress events) and conservativeness of risk measures.

Spectral risk measures allow to differently weight the observations in the tails by assigning more importance to the most extreme ones.

In particular, moving along the P&L distribution (e.g. moving from the smallest profit/loss in the tail to the biggest one) *spectral risk measures* allow to address the increasing risk-aversion of the CCP by assigning higher and higher weights.

In order to choose the *SRM* function two principles are taken into consideration:

1) the Risk Appetite Framework of the CCP must be satisfied, meaning that the results of the back-tests (both at ISIN and at Clearing Member portfolio level) must preserve the desired coverage level;
2) the anti-procyclicality policy of the CCP must be respected, meaning that possible increases in computed margin requirements must be checked against any anti pro-cyclicality thresholds.

Considered the two conditions above, a *spectral risk measure* that is both conservative (as per point 1 above) and not too punitive during crises (as per point 2) is chosen.

The chosen *spectral risk measure* works as follows:

denoting by $i = 1, \dots, n$ the tail events (where *1* corresponds to the smallest profit/loss and *n* to the biggest one), weights w_i are given by:

$$(1)\ \mathrm{w}_1 = \mathrm{x};$$

$$\mathrm{x}^{-1} = \frac{\mathrm{srm\_factor}^{(\mathrm{tail\_lenght}+1)} - \mathrm{srm\_factor} * (\mathrm{tail\_lenght}+1) + \mathrm{tail\_lenght}}{(1-\mathrm{srm\_factor})^2}$$

$$\mathrm{w}_2 = \mathrm{w}_1 + \mathrm{srm\_factor} * \mathrm{w}_1;$$

$$\mathrm{w}_3 = \mathrm{w}_2 + \mathrm{srm\_factor} * (\mathrm{w}_2 - \mathrm{w}_1);$$

$$\dots;$$

$$\mathrm{w}_n = \mathrm{w}_{n-1} + \mathrm{srm\_factor} * (\mathrm{w}_{n-1} - \mathrm{w}_{n-2}).$$

The weights must obviously sum to 1.:

Once the value of *x* is retrieved, all weights can be recursively retrieved. The weights are then applied to the observations in the tail in order to retrieve the *SRM Expected Shortfall.*

Consider as an example a 11 extreme events tail, obtained combining the *confidence level* to the *lookback period*, and a *SRM factor* equal to 1,35.

Table 30: Profits/losses tail observations

n	Profit/Loss
1	100
2	96
3	93

4	90
5	88
6	85
7	82
8	78
9	75
10	70
11	67

The 'plain' *Expected Shortfall* of such distribution amounts to 84 (average of the 10 observations).

The *SRM Expected Shortfall* would instead be defined through the following steps:

a) definition of the first weight $w_1=x$;
b) recursive definition of all the other weights $w_2, \ldots, w_{11}$:

n	Profit/Loss	Weight
1	100	0.29100
2	96	0.21267
3	93	0.15465
4	90	0.11167
5	88	0.07983
6	85	0.05625
7	82	0.03878
8	78	0.02584
9	75	0.01626
10	70	0.00916
11	67	0.00390
		Sum of weights = 1

c) computation of the weighted profits/losses by multiplying each observation by its weight:

n	Profit/Loss	Weight	Weighted Profit/Loss
1	100	0.29100	29.10
2	96	0.21267	20.42
3	93	0.15465	14.38
4	90	0.11167	10.05
5	88	0.07983	7.03
6	85	0.05625	4.78
7	82	0.03878	3.18
8	78	0.02584	2.02
9	75	0.01626	1.22
10	70	0.00916	0.64
11	67	0.00390	0.26
		Sum of weights = 1	

d) computation of the *SRM Expected Shortfall* summing all weighted profits/losses (amounting to 93.07).

The *SRM Expected Shortfall* yields a more conservative result with respect to the 'plain' *Expected Shortfall*, as expected.