EURONEXT QUANTITATIVE RESEARCH REPORT

Better trading at the close thanks to market impact models

25 January 2021

In 2020, closing auctions market share represented more than 20% of European consolidated volumes. Surprisingly, almost no publicly available market impact model on closing auctions is available, although the continuous market impact has been extensively studied. As a market operator we share with all market participants the findings of our unique dataset. In particular we evidence four main results on closing auctions:

- We highlight that indicative prices overreact on average during Call phases and that this pattern is explained by the temporal imbalances of Market and Limit orders (see Figure 13, p13 and Figure 16, p15).
- We describe the instantaneous impact and its subsequent decay following a Market order submission (see Figure 19, p18). We show that early order submissions have less price impact than later submissions (see Figure 23, p21).
- We establish a market impact model on Close for Market orders. We show that for a given trade size, the resulting market impact on Close is two to three times smaller than it is for continuous trading (see Figure 25, p26 and Figure 26, p27). This comes as no surprise as the Close represents the most liquid event in equity markets.
- Lastly we raise the question of the internalisation of Market orders and its adverse consequences on auction volatility, as shown by the increasing standard deviation of the Jump on Close when the share of matched Market orders decreases (see Figure 30, p31 and Figure 32, p32).

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INTRODUCTION

The strong rise in the market share of closing auctions observed from 2008 to 2020 has attracted much interest from market participants, as well as from regulators and academics. While most commentators attribute this rise to the growing share of ETFs, very few publicly available research papers study the market impact of trades executed at the Close (for more details see our review of literature in the Appendix).

As best execution enforcement strengthens across all investor types, we believe that the cost of trading at the Close is a key driver in understanding the strong increase in closing auction market share. We also believe that the recent growing popularity of alternative mechanisms for trading on Close raises new systemic questions concerning the quality of the prices in closing auctions.

Today, all too often, only the most quantitatively advanced market participants have the inhouse resources and the available data to build internal market impact models. This allows them to make the best choices regarding their execution policy. We believe it is our mission as the leading pan-European exchange to set a level playing field for all, and support all types of investors in making their best execution choices. Our publicly available Quantitative Research papers aim to help investors analyse where they can find best execution, based on all the available data.

This paper is structured in five independent parts:

- Part 1 provides some basic statistics about the stock universe and the period we consider, as well as some key features of order submission on Close.
- Part 2 looks at the indicative volume and price profiles during the auction Call phase and explains the patterns we observe in participants' behaviour.
- Part 3 focuses on the formation of market impact during the Call phase. In particular we study the instantaneous market impact following an order submission as well as the decay after the initial impact to the end of the uncrossing.
- Part 4 tackles the question of the relative cost of trading at the closing auction versus during the continuous market. Therefore we first review the cost of continuous trading on equities based on public market impact models. We then establish that the cost of continuous trading greatly exceeds the cost of trading at the closing auction.
- Part 5 addresses the systemic consequences of order internalisation on the quality of closing auctions. In particular we show that a smaller share of matched Market orders (more likely to be internalised) is associated with larger auction volatility, which is detrimental to all market participants as closing prices are the major valuation reference for equities.

UNIVERSE AND KEY FEATURES

STOCKS AND TIME PERIOD

We considered 160 stocks from the three largest Euronext cash equity markets at the end of 2019: Belgium, France and the Netherlands. Within these countries we considered constituent stocks from both the Large capitalisation indices and the associated Mid capitalisation indices as displayed in Table 1, we considered the stock components as of 4 November 2019.

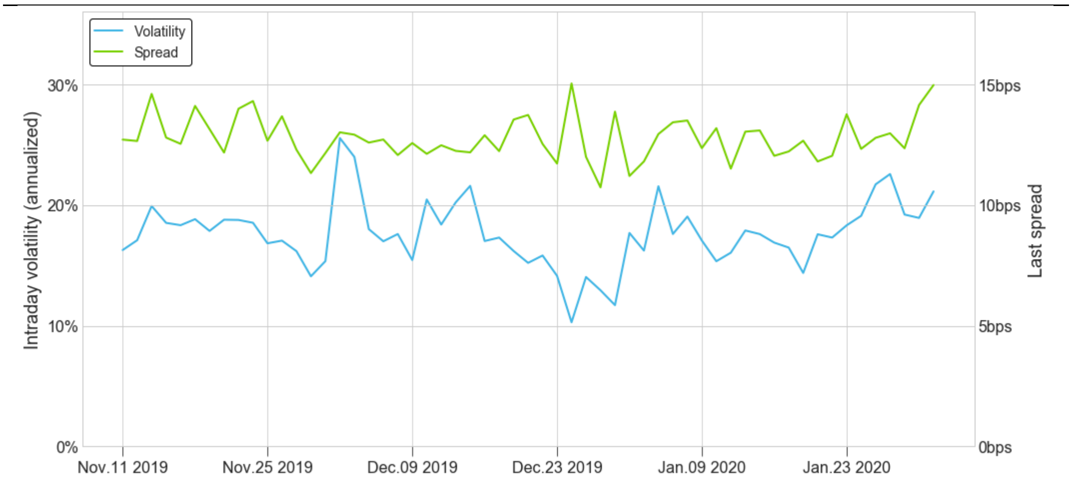
Table 1: Stock universe

Index	Number of components	Market cap	Country
CAC 40®	40	Large	France
AEX®	25	Large	Netherlands
BEL 20®	20	Large	Belgium
CAC Next 20®	20	Mid	France
AMX®	25	Mid	Netherlands
BEL Mid®	39	Mid	Belgium
ALL	**160**	**84 Mid & 76 Large**	**3 Euronext countries**

Source: Euronext

We considered 3 months of data from 4 November 2019 to 31 January 2020. Thus, we reviewed 62 trading days prior to the market volatility spikes observed after February 2020 due to the outbreak of the Covid-19 pandemic. As observed in Figure 1, in late 2019 and early 2020 the intraday volatility (blue line) ranged between 10% and 25%. The average last bid-ask spreads of the continuous phase in our stock universe (see green line) fluctuated between 12 bps and 15 bps.

Figure 1: Spread and intraday volatility timeline



Average across stocks. Intraday volatility computed with Garman Klass estimator.
Dataset Daily (filtered)
Source: Euronext

Our diverse stock universe comprises Small caps with daily turnover below €1 million present in the Midcap Belgian index, as well as Large caps with daily turnover larger than €100 million present in the CAC 40® and AEX® indices, as displayed in Figure 2. Similarly, average last spreads range between 3.5 bps and almost 40 bps for the smallest stocks in the BEL Mid® index as shown in Figure 3.

Figure 2: Turnover across stocks

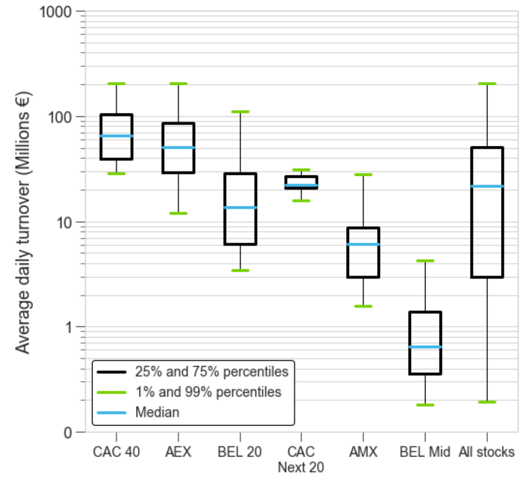
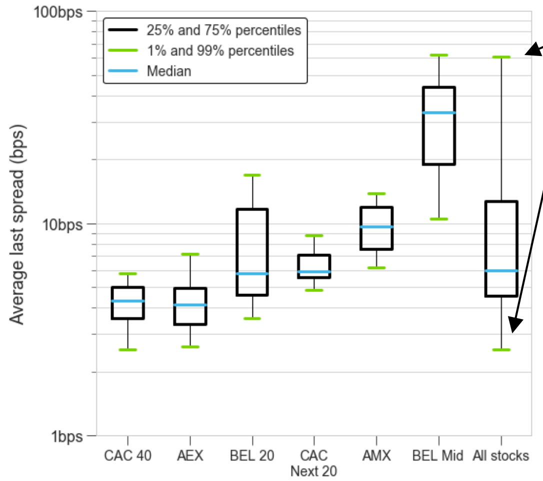


Figure 3: Spreads across stocks



In our stock universe most spreads range between 2.5 bps and almost 60 bps

Dataset Daily (filtered)
Source: Euronext

DATASET

In our paper we refer to four imbricated datasets based on the 160 stocks mentioned above, from 4 November 2019 to 31 January 2020.

- The largest dataset is called ‘Updates’. It contains all the events in the closing orderbook (during the Call phase and the closing uncrossing), which amounts to 9,439,871 updates.
- The second dataset is called ‘Last Modif.’. It contains 5,835,035 events which correspond to the last modification of an order, that is either a ‘New’ order (if the order was not modified or cancelled afterwards), a ‘Modify’ or a ‘Cancel’.
- The third dataset is called ‘Trades’. It contains 1,706,786 elementary trades executed at the closing uncrossing.
- The fourth and final dataset is called ‘Daily’. It contains daily aggregated metrics over the 9,920 auctions in the study.

For technical reasons, in our study, we used additional filters on the four datasets above. All filters are described specifically in Table 2. In all of the charts in the report the dataset used is defined, as well as all the filters applied.

Table 2: Our four datasets: filtered and unfiltered

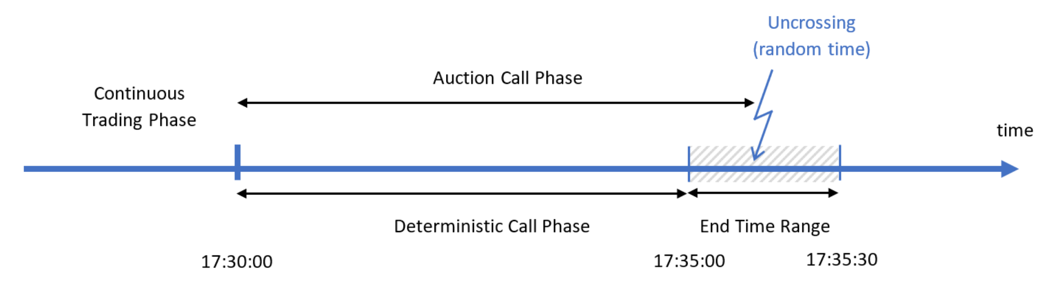
Type Names	Events 'Updates'	Events 'Last Modif.'	Executions 'Trades'	Aggregates 'Daily'
Unfiltered	**9,439,871**	**5,835,035**	**1,706,786**	**9,920**
Non-zero uncrossing volume	Active		Active	Active
Orders executed during the uncrossing only			Active	
Order size between 0.1% and 10% of the daily volume			Active	
Order size between 0.1% and 5% of the daily volume		Active		
Indicative price available	Active	Active		
Market orders only		Active		
Not modified nor cancelled		Active		
Non-zero Jump on Close	Active			
Filtered	**9,259,902**	**258,542**	**763,268**	**9,858**

Source: Euronext

CLOSING AUCTION CHARACTERISTICS

Orders destined for the closing auction can be submitted during the continuous trading day, but book building only starts at the beginning of the Call phase. Since late 2015 and the introduction of a random end time at the end of the Call phase as represented in Figure 4, the features of the closing auction Call phase have not changed. The random time introduction prevents participants with better latency from taking advantage of slower participants as the end of the Call phase approaches.

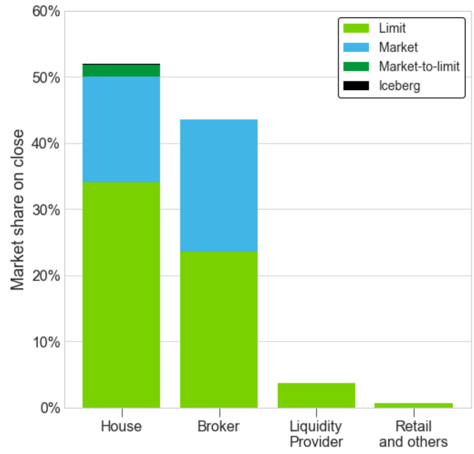
Figure 4: Closing auction timeline



Source: Euronext

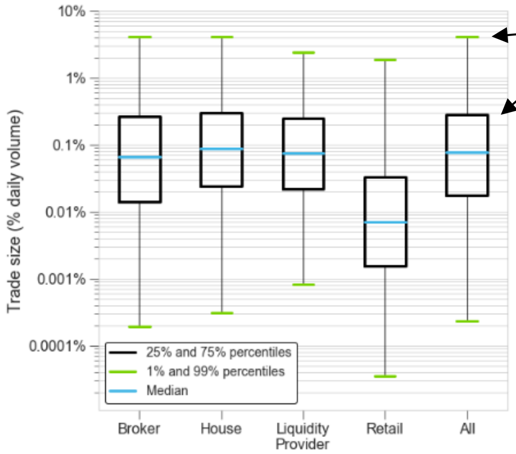
Market participants are split into 4 categories: House, Broker, Liquidity Providers and Retail. As required by MiFID, participants are requested to declare the type of their flows to exchanges. 'House' represents trading for proprietary trading desks or trading firms, 'Broker' regroups agency flows, 'Liquidity Providers' refers to flow from specific market participants that benefit from the liquidity providers fee scheme, and 'Retail' encompasses retail flows executed by retail brokers.

Figure 5: Market share on Close by order type and participant type



Dataset Trades (unfiltered)
Source: Euronext

Figure 6: Trade sizes on Close



Dataset Trades (unfiltered)
Source: Euronext

Only 1% of trades are larger than 4% of Daily Volume and 25% of them are larger than 0.3%

Unlike during the continuous phase, on closing auctions, House and Broker represent 95% of turnover while Liquidity Provider flows only amount to 4% of turnover, as displayed in Figure 5. While four types of orders are available at the closing auction, Limit and Market orders represent more than 95% of all trades at the close.

As displayed in Figure 6, trade sizes are comparable across different market participants; only retail flows display smaller average trade sizes than other participants. 25% of trades are larger than 0.3% of Daily Volume (DV) and only 1% are greater than 4% of DV.

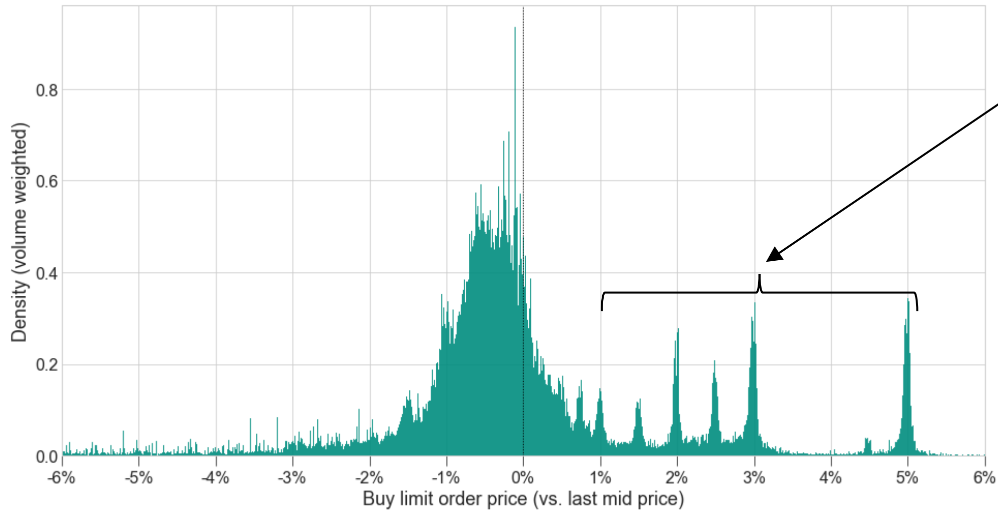
Remark: In this note trade sizes on a given stock will be expressed in % of the daily volume on the primary market. We first use this normalisation in order to compare trade sizes across different stocks, but also later on in order to compare market impact at the closing auction as well as during the continuous market.

KEY FEATURES OF THE ORDER SUBMISSION ON CLOSE

In order to study Limit orders, we express limit prices relative to the last mid price associated with the last state of the primary market continuous orderbook. More precisely, we use a normalised limit price $\frac{P_{\text{Limit}} - P_{\text{Last Mid}}}{P_{\text{Last Mid}}}$, where P_{Limit} is the price limit of a given order and $P_{\text{Last Mid}}$ is the last mid price of the continuous phase.

Furthermore, we display the Limit orders price distribution, in Figure 7, we only show Buy Limit orders, having verified that the distribution of Sell limit prices is symmetrical with the distribution of Buy limit prices.

Figure 7: Average Limit order price distribution on Close



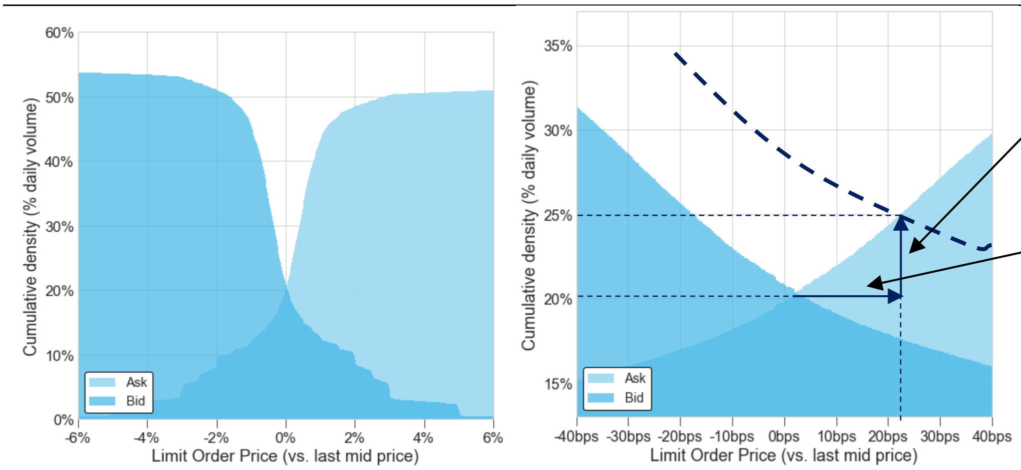
Dataset Last Modif. (unfiltered)
Filter: Buy Limit orders still in the order book at the end of the Call phase only (3,106,438 orders)
Source: Euronext

At several round values, +1%, +2%, +3%, +5%, as well as for +1.5% and +2.5%, large volume spikes of Limit orders are observed

We observe in Figure 7 that Limit orders with limit prices below the last continuous mid price are almost distributed normally. These orders correspond to opportunistic Buy orders that are only triggered if the auction price settles below the last continuous price. In contrast the right-hand side of the distribution displays several modes.

At several round values (1%, 2%, 3%, 5%) as well as for 1.5% and 2.5%, large volume spikes of orders are observed. These correspond to aggressive participants who prefer to send a Limit order with a very high limit price rather than sending a Market order.

Figure 8: Liquidity available on Close (the right panel is a zoomed version of the left panel)



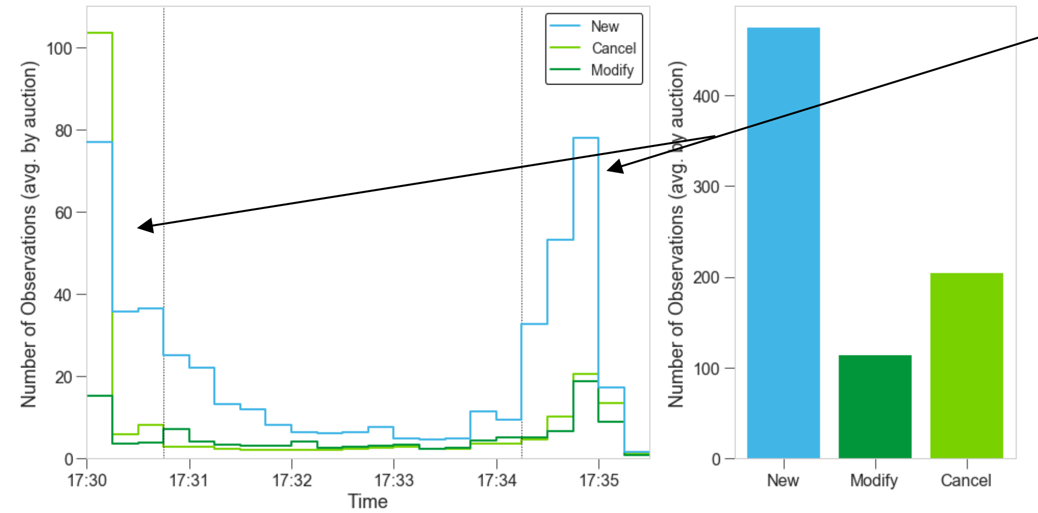
Dataset Last Modif. (unfiltered)
Filter: Limit orders still in the order book at the end of the Call phase only (3,106,438 orders)
Source: Euronext

An additional +5% of the Daily Volume (from 20% to 25% of the Daily Volume) is available for sale at a price +23 bps above the initial estimated settlement price

When both cumulated Buys and Sells volumes are represented in Figure 8 (left-hand side), we observe the classical overlapped auction orderbook shape. In order to account for the unmatched liquidity available at the closing auction we have represented in the right side of Figure 8 a zoom of the orderbook.

We show that on average, for an unchanged Ask side (Sell Limit orders, in light blue), an additional +5% of the Daily Volume (from 20% to 25% of the daily volume) is available for sale at a price +23 bps above the initial estimated settlement price (see dotted line in Figure 8 right-hand chart). This observation already provides us with an empirical assessment of the available liquidity at the Close.

Figure 9: Orders submission profile upon the Call phase (all types of orders are grouped together in these charts)



Observations grouped by 15 second bucket intervals.
Dataset Updates (unfiltered)
Source: Euronext

Most orders are submitted either at the start or at the end of the Call phase

Most of the orders are submitted either at the start or at the end of the Call phase as displayed in Figure 9. These submissions are split into three categories: New orders, Modifications of the size of existing orders, and Cancellations.

More precisely we display two vertical grey lines at 17:30:45 and at 17:34:15 to highlight the fact that over 75% of the newly submitted orders and 85% of the Cancellations take place during either the first or the last 45 seconds of the deterministic Call phase (lumped with the orders during the random time span). In what follows we will refer to the period from 17:34:15 to the uncrossing simply as 'last 45 seconds of the deterministic Call phase'.

ESTIMATED VOLUME AND PRICE PROFILE DURING THE CALL PHASE

INDICATIVE CLOSING VOLUME

During the Call phase, at every change in the closing auction orderbook an indicative closing volume as well as an indicative closing price are displayed publicly. In order to study the indicative volume during the closing auction Call phase as well as across stocks we use the following normalisation:

$$\text{Indicative Volume (\% closing volume)} = 100 \times \frac{Q_{\text{indic.}}}{Q_{\text{Close}}},$$

with $Q_{\text{indic.}}$ the indicative volume and Q_{Close} the actual closing volume for one stock.

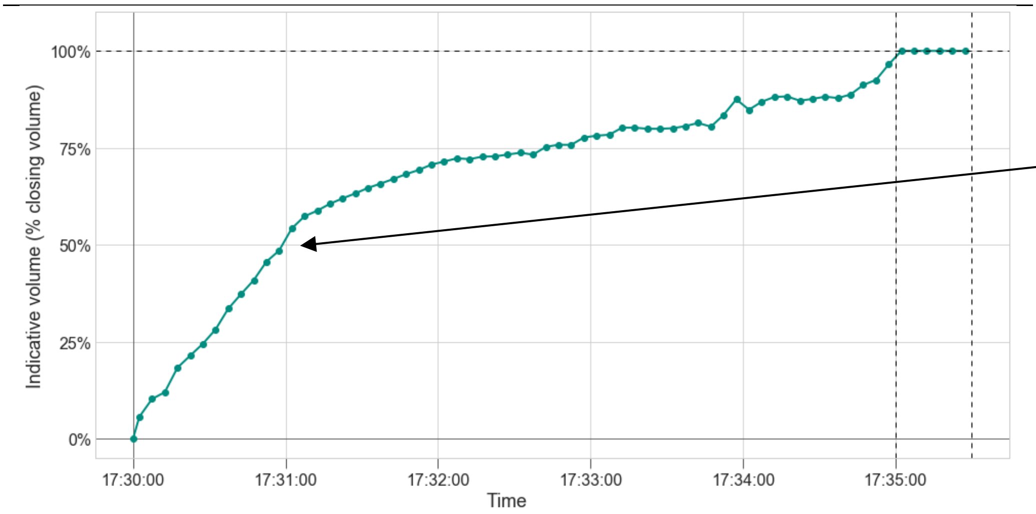
MAIN FEATURES OF INDICATIVE VOLUME PROFILE

Indicative volume profile over time

Using the above normalisation, we aggregate the Indicative Volume profiles for 9,920 auctions, using more than 9.4 million updates (see ‘Updates’ dataset in Table 2). Plotting a median estimate for every 5 second interval, we trace the median Indicative Volume profile as displayed in Figure 10.

Overall the Indicative Volume increases with time. We observe that after the first minute of the Call phase (17:31:00), the Indicative Volume already represents more than 50% of the closing volume, and that halfway through the Call phase (17:32:30), the Indicative Volume reaches almost 75% of the final closing volume. Lastly, at the end of the deterministic time span (17:35:00), 100% of the closing volume is already reached, showing that only a minority of the orders contributing to the auction are displayed during the random end time.

Figure 10: Indicative Volume profile during the closing Call phase



After the first minute of the Call phase, the Indicative Volume already represents more than 50% of the closing volume

Each dot represents the y-axis median computed for each 5 second interval of the Call phase.
Dataset Updates (filtered)
Source: Euronext

Indicative Volume profile across stocks

We then trace Indicative Volume profiles across Large and Mid caps (as defined in Table 1). We observe in Figure 11 that Indicative Volume profiles for Mid caps (green line) are rising less quickly than Indicative Volume for Large caps: at 17:32:30 they amount to only 62% of the closing volume versus 76% for Large caps (blue line). Consistently, stocks with larger spreads (blue line in Figure 12) display less steep Indicative Volume curves than stocks with smaller spreads (green line).

Figure 11: Indicative Volume profile for different market capitalisations

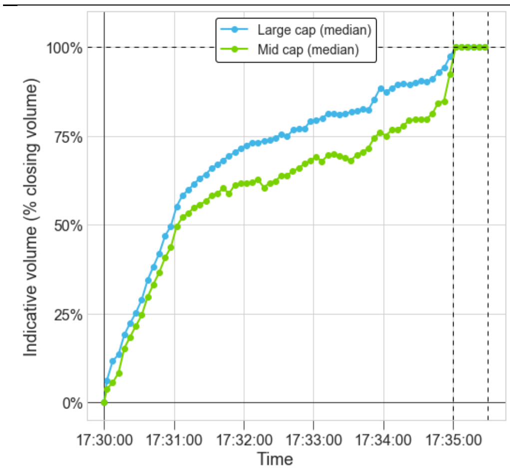
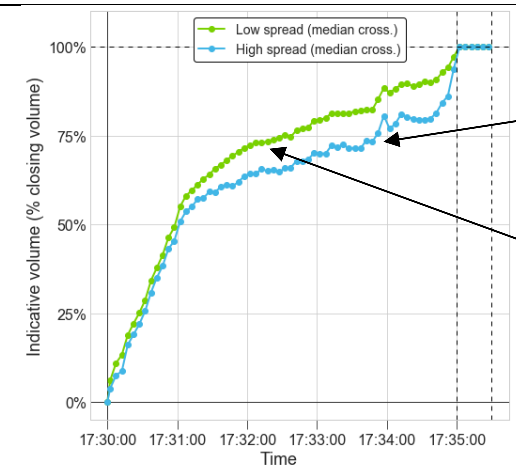


Figure 12: Indicative Volume profile on stocks with larger and smaller spreads



Stocks with larger spreads display less steep Indicative Volume curves than stocks with tighter spreads

Each dot represents the y-axis median computed for each 5 second interval of the Call phase.
'cross' stands for: At each date our stock universe is split evenly into 50% larger and smaller spread buckets.
Dataset Updates (filtered)
Source: Euronext

INDICATIVE PRICES AND JUMP ON CLOSE

Indicative closing prices enable market participants to estimate the future closing price based on the current state of the closing auction orderbook.

First, we recall the Jump on Close formula, which is often defined as:

$$\text{Jump on Close} = side \times \frac{P_{\text{Close}} - P_{\text{Last Mid}}}{P_{\text{Last Mid}}},$$

where $side = +1$ for a Buy order and -1 for a Sell order.

Then, in order to analyse average price profiles during the Call phase we define the Indicative Jump on Close at every orderbook update as:

$$\text{Indicative Jump on Close} = side \times \frac{P_{\text{indic.}} - P_{\text{Last Mid}}}{P_{\text{Close}} - P_{\text{Last Mid}}},$$

when $P_{\text{Last Mid}} \neq P_{\text{Close}}$.

Above, $P_{\text{indic.}}$ designates the indicative closing price, P_{Close} is the actual closing price and $P_{\text{Last Mid}}$ is the last mid price observed on the primary market during the continuous trading phase.

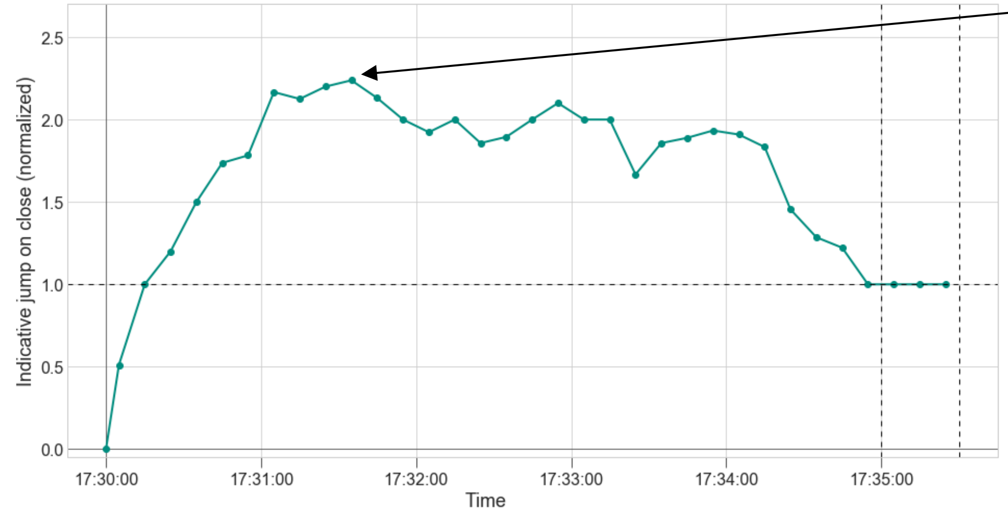
The benefit of this normalisation is that the Indicative Jump on Close takes the value 0 at the start of the Call phase and +1 at the uncrossing of the closing auction, regardless of the closing auction price.

INDICATIVE PRICES OVERREACT DURING THE CALL PHASE

We then plot on all of the 9,858 closing auctions of our dataset, for every 10 second bucket, the median Indicative Jump on Close across all stocks.

On average we observe in Figure 13 that indicative prices overreact, as shown by the 2.2 value of the maximum of the Indicative Jump on Close observed at 17:31:30. This means that on average an indicative price change larger than 2.2 times the realised Jump on Close is observed during the Call phase. After the first 1 minute 30 seconds the indicative price reverts (slowly at first, and then more abruptly after 17:34:00) towards the final closing price.

Figure 13: Price formation during the closing Call phase



Each dot represents the y-axis median computed for each 10 second interval of the Call phase.
Dataset Updates (filtered)
Source: Euronext

Indicative prices overreact on average, as shown by the 2.2 value of the maximum of the Indicative Jump on Close observed at 17:31:30.

This overreaction is amplified on large stocks compared to smaller stocks, as displayed in Figure 14. Mid caps (green line) display a smaller Indicative Jump on Close than those with larger capitalisations (blue line).

Figure 14: Price profile for different market capitalisations

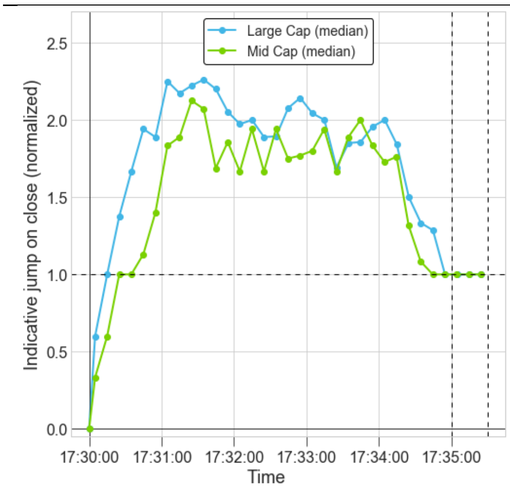
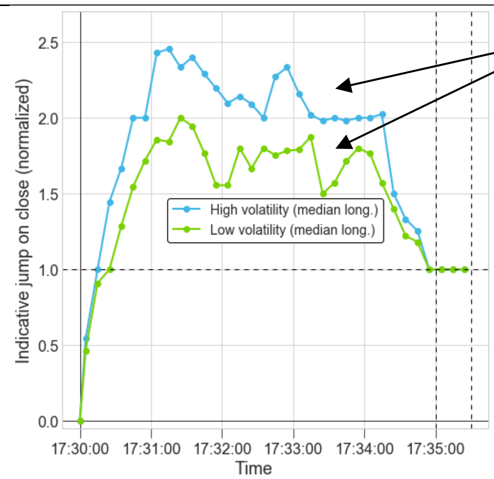


Figure 15: Price profile dependency on intraday volatility



Each dot represents the y-axis median computed for each 10 second interval of the Call phase.
'long' stands for: For each stock we split daily observations into 50% with larger and smaller volatility buckets.
Dataset Updates (filtered)
Source: Euronext

High volatility days show a larger overreaction in indicative prices than low volatility days

For a given stock, a rise in intraday volatility translates into a larger overreaction of the Indicative Jump on Close as observed in Figure 15. Here, for each stock in our universe we have divided auctions into two categories: high volatility and low volatility (intraday). This way, the same stocks are present in both the high volatility (blue line) and the low volatility (green line) categories. We clearly observe that the high volatility days show a larger overreaction, as shown by the blue line above the green line.

ORDER SUBMISSION IMBALANCE DURING THE CALL PHASE

Useful normalisations

To investigate the pattern in the order submission that triggers the average price overreaction, we first need to define the adjusted imbalance that we will represent for both Market and Limit orders. In our definition, positive Adjusted Order Imbalances will correspond to orders whose contribution to price changes will be of the same sign as the overall Jump on Close.

- For Market orders we define during each time interval of the Call phase:

$$\text{Adjusted Market Order Imbalance} = sign(\text{Jump on Close}) \times \frac{Q_{\text{New Market Buy}} - Q_{\text{New Market Sell}}}{Q_{\text{Close}}},$$

  where $Q_{\text{New Market Buy}}$ and $Q_{\text{New Market Sell}}$ represent the volume in shares of the new Buy and Sell Market orders submitted during a given timeframe.

- For each new Limit order, we only consider Executable Limit Orders, that is Limit orders whose limit prices were superior to the closing prices for Buys and inferior to the closing prices for Sells.

  We now define:

$$\text{Adjusted Limit Order Imbalance} = sign(\text{Jump on Close}) \times \frac{Q_{\text{New Exec.Limit Buy}} - Q_{\text{New Exec.Limit Sell}}}{Q_{\text{Close}}},$$

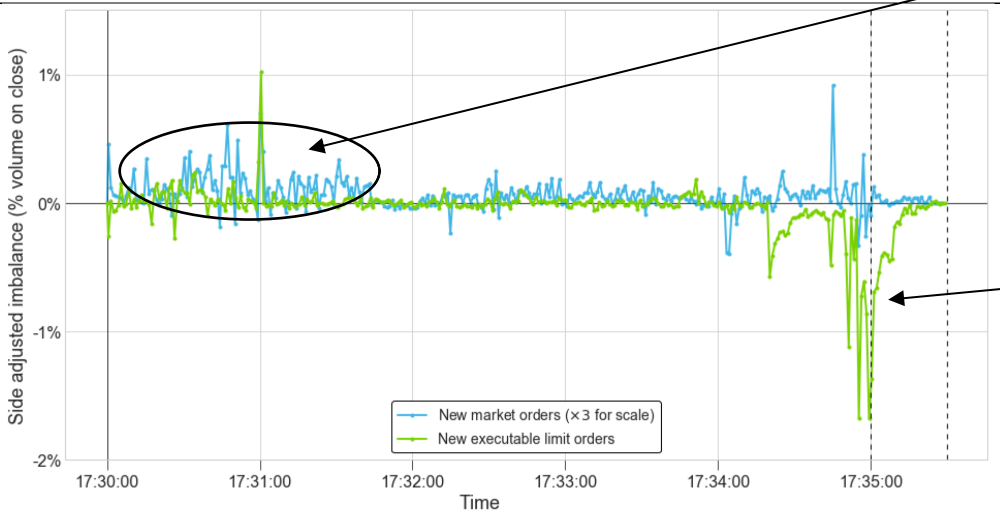
  where $Q_{\text{New Exec.Limit Buy}}$ and $Q_{\text{New Exec.Limit Sell}}$ represent the volume in shares of the new Buy and Sell Executable Limit Orders submitted during a given timeframe.

The imbalance of Market and Limit orders explain the indicative price overreaction during the Call phase

During the first 90 seconds, a positive Adjusted Market Order Imbalance (blue line) is clearly observed on average as well as a slightly positive Adjusted Limit Order Imbalance (green line), as displayed in Figure 16. These imbalances account for the overreaction observed on average during the first 90 seconds of the Call phase (as shown in Figure 13).

During the last 60 seconds of the deterministic Call phase, we observe a very strong negative Adjusted Limit Order Imbalance (see green line in Figure 16) that coincides with the average reversion of indicative prices toward the closing price. This analysis suggests that the negative Adjusted Limit Order Imbalance causes the average price reaction at the end of the Call phase as depicted in Figure 13, although a strict causal analysis would be required to confirm this claim.

Figure 16: Imbalance in Market orders as well as for executable Limit orders



Each dot represents the y-axis average computed for each second of the Call phase.
Dataset Updates (filtered)
Filter: New orders only (4,622,332 orders)
Source: Euronext

The market order imbalance (blue) accounts for the overreaction observed on average during the first 90 seconds of the Call phase

At the end of the Call phase, the opposite Limit Order Imbalance causes the average price reversion

THE FORMATION OF THE MARKET IMPACT ON CLOSE DURING THE CALL PHASE

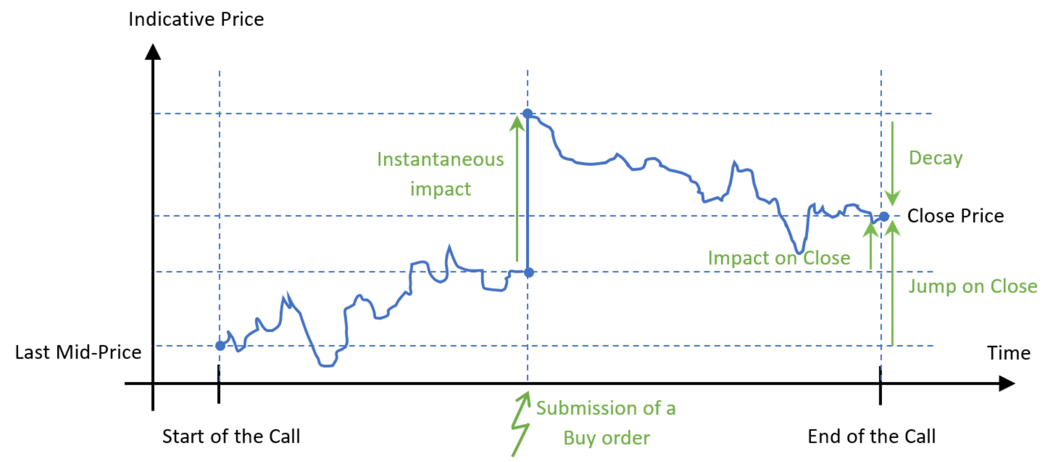
After analysing the market impact on Close, we now continue our investigation by exploring the process of the market impact itself during the closing auction Call phase. In particular we will look at the immediate effect on indicative prices of the submission of an order, as well as its effect until the end of the closing auction. This will help understand the role of the timing of order submission during the Call phase.

METRICS AND TIMEFRAME

Breaking down the Market Impact on Close into sub-components

For any elementary order submitted during a closing auction Call phase, its Impact on Close (as defined below) can be viewed as the sum of two sub-components: the Instantaneous Impact and its associated Decay (Figure 17).

Figure 17: Instantaneous Market Impact, Decay, Impact on Close and closing auction



Source: Euronext

More precisely:

The 'Instantaneous Impact' measures the immediate market impact on the indicative closing price:

$$\text{Instantaneous Impact} = side \times \frac{P_{\text{indic. after submission}} - P_{\text{indic. before submission}}}{P_{\text{Last Mid}}}$$

where $P_{\text{indic. after submission}}$ and $P_{\text{indic. before submission}}$ represent the last indicative price before the order submission and the first indicative price after the order submission.

The 'Decay' characterises the ensuing indicative price change measured between the Instantaneous Impact and the end of the closing auction:

$$\text{Decay} = side \times \frac{P_{\text{Close}} - P_{\text{indic. after submission}}}{P_{\text{Last Mid}}}$$

We can therefore define the 'Impact on Close' which measures the impact of a newly submitted order from its submission during the Call phase to the closing uncrossing:

$$\text{Impact on Close} = side \times \frac{P_{\text{Close}} - P_{\text{indic. before submission}}}{P_{\text{Last Mid}}}$$

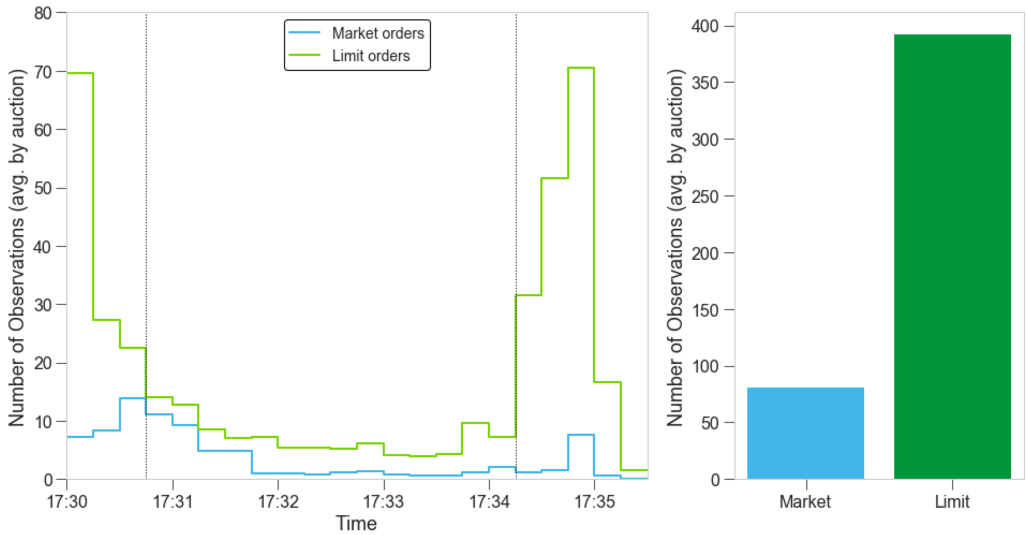
Type of orders and time period considered in our analysis

For the sake of simplicity in this section, we will only consider Market orders for the study of the market impact, since Limit orders are more subtle: the level of the limit price must be taken into account, which makes the formation of the market impact more complex to understand.

Among Market orders we will only consider 'New' orders that have not been modified nor cancelled after their initial submission, so that our results are not distorted by new events happening on existing orders after their initial submission. The Market orders that are neither cancelled nor modified still represent 32% of newly submitted orders (258,542 observations, see 'Last Modif.' filtered dataset Table 2, p7).

In order to study the role of timing in market impact we will highlight two specific timeframes in the Call phase: the first 45 seconds after 17:30, and the last 45 seconds of the deterministic time span (see Figure 18, vertical grey lines). During the first period, 37% of Market orders are submitted, while during second only 18% of Market orders are submitted.

Figure 18: Submission profile of New orders during the call phase



Dataset Updates

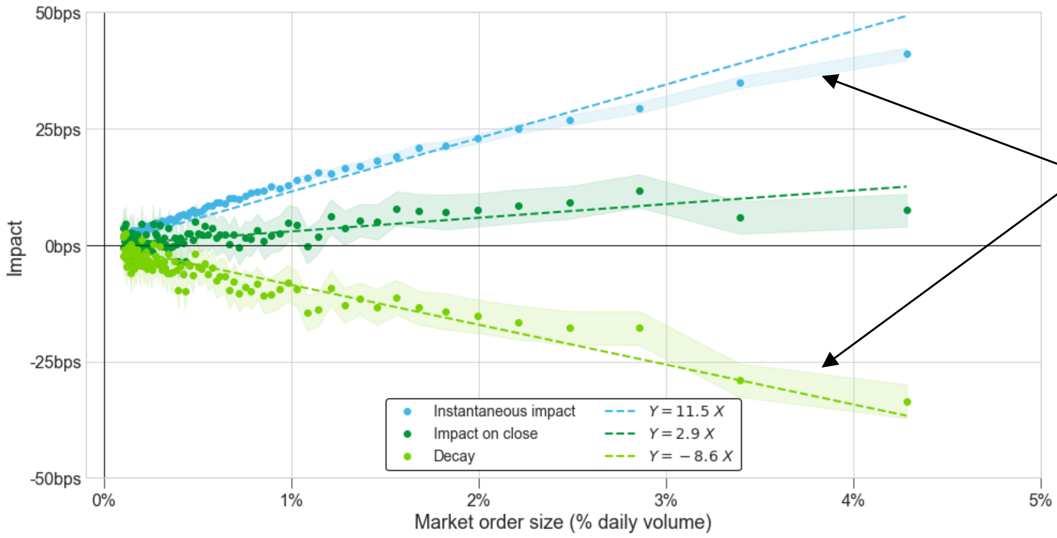
Filter: New orders only (4,704,069 orders among which 258,542 Market orders unmodified and not cancelled).

Source: Euronext

THE INSTANTANEOUS IMPACT, THE DECAY AND THE IMPACT ON CLOSE

As already discussed, the Impact on Close is the sum of the Instantaneous Impact and the Decay. Using our 'Last Modif.' dataset, and after removing the 1% largest market impact in absolute value, we plot in Figure 19 the Instantaneous Impact of newly submitted Market orders in blue, the Decay in light green, and the resulting Impact on Close in dark green. We observe that the Instantaneous Impact increases almost linearly with the Market order size. Likewise, the Decay is amplified with rising Market order sizes. The slope of the Decay amounts to −75% of the slope of the initial Instantaneous Impact. This shows that a large mitigation of the initial impact comes from the reaction of the market participants after the immediate rise of the indicative closing price. Overall, the Impact on Close increases linearly with the Market order size.

Figure 19: Decomposition of the market impact on Close



Each dot represents the y-axis average computed on a percentile of the x-axis distribution.
Dataset Last Modif. (filtered)
Filter: $|\text{instant. impact}| \leq q_{99\%} = 128\text{bps}$ (255,956 orders)
$+\ |\text{impact on close}| \leq q_{99\%} = 534\text{bps}$ (254,028 orders),
$+\ |\text{decay}| \leq q_{99\%} = 534\text{bps}$ (253,989 orders)
Source: Euronext

75% of the initial Instantaneous Impact is largely mitigated by the Decay. Overall, the resulting Impact on Close (dark green) increases linearly with the Market order size

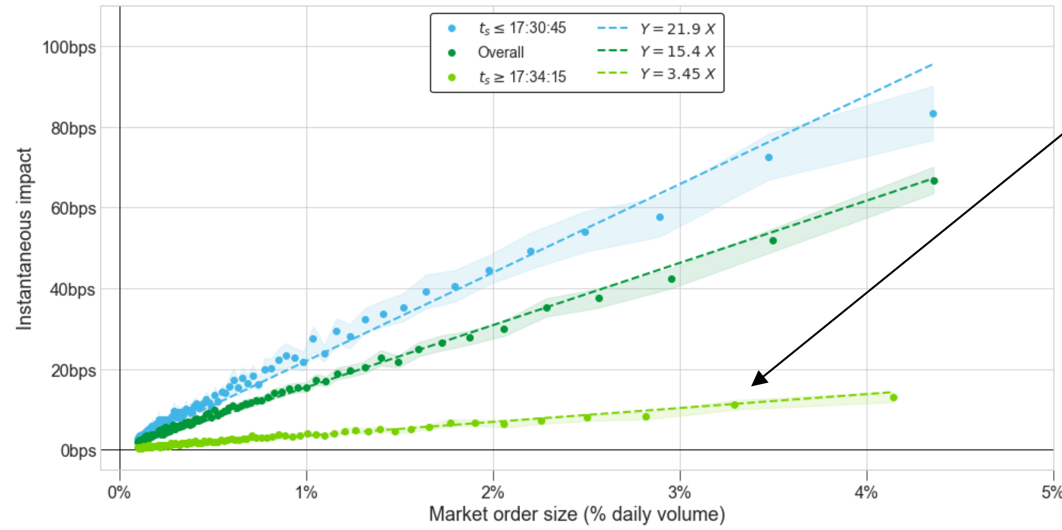
WHEN TO SUBMIT MARKET ORDERS DURING THE CALL PHASE?

Instantaneous Impact lessens for later submissions

In Figure 20 we have plotted the Instantaneous Impact versus the order size for different submission time ranges. Early Market order submissions in the first 45 seconds of the Call phase (in blue) display a larger Instantaneous Impact than overall submissions (in green) and later submissions (in the last 45 seconds of the call phase) in light green.

This comes as no surprise as Instantaneous Impact is a direct consequence of the liquidity of the orderbook: the more liquid the orderbook, the smaller the impact of a given trade. Since the indicative volume increases with time (as already discussed), it makes sense that a smaller Instantaneous Impact is observed for later submissions.

Figure 20: Instantaneous Impact for different Market order submission times



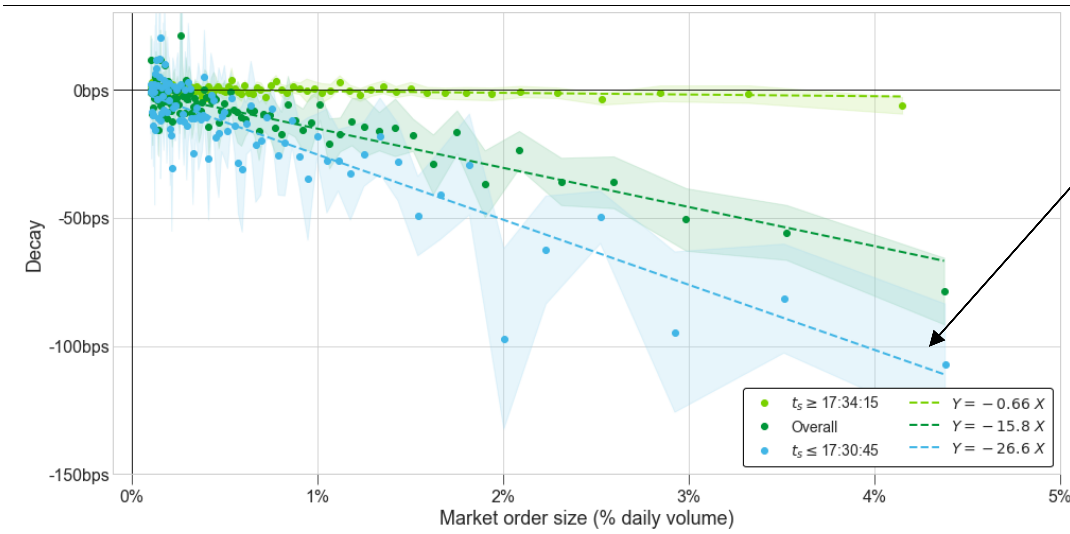
Each dot represents the y-axis average computed on a percentile of the x-axis distribution.
Dataset Last Modif. (filtered)
Filter: instant. impact $\leq q_{99.9\%} = 916$bps (236,032 orders)
Source: Euronext

A smaller Instantaneous Impact is observed for later submissions

An amplified Decay is observed with earlier submissions

As already seen in Figure 19, 75% of the Instantaneous Impact is mitigated by the Decay. It is therefore expected that a greater Instantaneous Impact will give way to an amplified Decay. In Figure 21, we have plotted the Decay versus the order size for different submission time ranges. Consistently, the Decay following an earlier submission (in light blue) is on average more negative than the Decay following a later submission (light-green line).

Figure 21: Decay for different Market order submission times



Each dot represents the y-axis average computed on a percentile of the x-axis distribution.
Dataset Last Modif. (filtered)
Filter: $|\text{decay}| \leq q_{99.9\%} = 6{,}544$bps (236,035 orders)
Source: Euronext

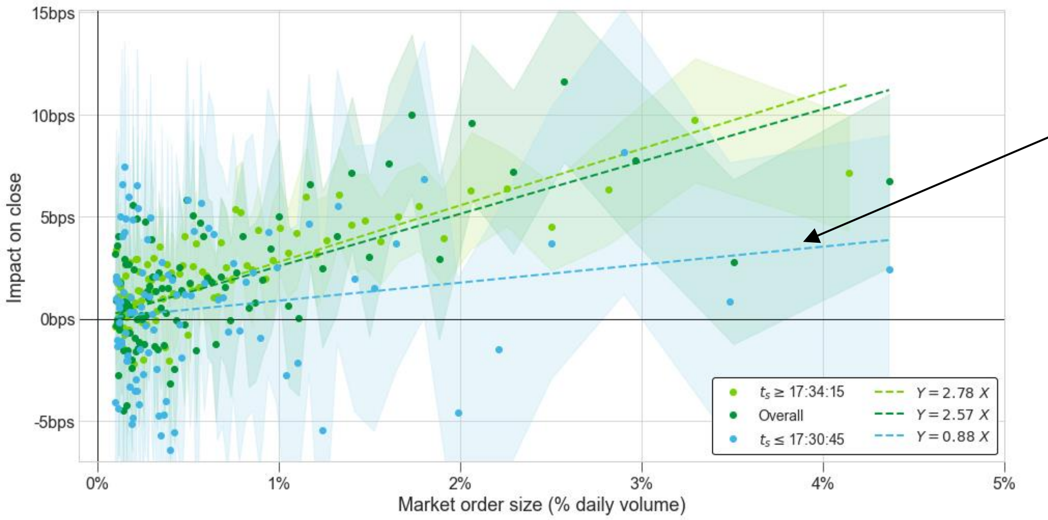
A stronger Decay is observed after an earlier submission

The overall Impact on Close is mitigated for later submissions

As seen above, we observe two opposite phenomena affecting the time dependency of the Impact on Close: Instantaneous Impact is greater for earlier trades but so is the subsequent opposite Decay. Further investigation is thus needed to conclude whether earlier trades have greater or smaller impact on the closing price.

In Figure 22 we plot the corresponding Impact on Close of earlier and later New Market order submissions. We observe that earlier submissions in light blue display a much smaller Impact on Close than later submissions (in green) for a given order size. This result is evidenced by the relative slopes of the Impact on Close versus the Market order size of 2.78 for orders posted after 17:34:15 and of 0.88 for orders posted before 17:30:45 (see equations caption in Figure 22). This shows that an earlier submission of a Market order has less impact than a later submission. More precisely, despite greater Instantaneous Impact, the subsequent Decay resulting from the reaction of other participants renders the Impact on Close smaller for earlier Market order submissions than for later ones. This shows the key role of the arbitragers, who have more time to mitigate Instantaneous Impact when orders are submitted earlier in contrast to late submissions.

Figure 22: Impact on Close for different Market order submission times



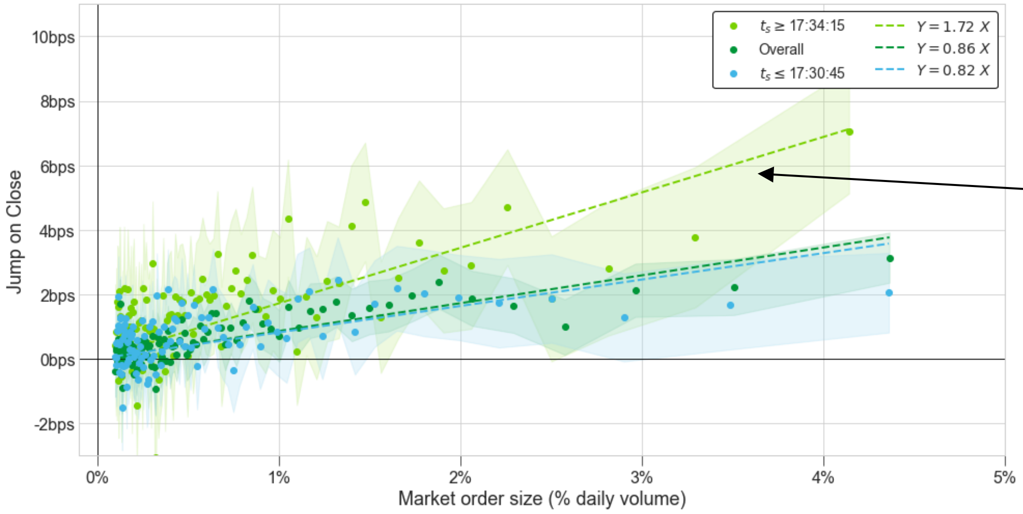
The Impact on Close is smaller for earlier Market order submissions

Each dot represents the y-axis average computed on a percentile of the x-axis distribution.
Dataset Last Modif. (filtered)
Filter: impact on close $\leq q_{95\%} =$ 254bps (245,614 orders)
Source: Euronext

Jump on Close is also smaller in cases of early submission

In order to back up our earlier finding, we separate into two buckets the orders of our 'Last Modif' dataset based on their submission time, and then plot their corresponding Jump on Close in Figure 23. We observe that trades submitted earlier (in light blue) result in a smaller Jump on Close than later trades (in light green). This finding corroborates our earlier result that earlier submissions result in smaller Market Impact on Close than later submissions, and thus reduce the cost of trading at the Close.

Figure 23: Jump on Close in bps by time of submission



Each dot represents the y-axis average computed on a percentile of the x-axis distribution.
Dataset Last Modif. (filtered)
Source: Euronext

The Jump on Close is larger for later Market order submissions

MARKET IMPACT ON CLOSE IS SMALLER THAN CONTINUOUS MARKET IMPACT

When traders are considering whether to execute at the Close or during the continuous trading phase, one of the main drivers in their decision is the expected cost of trading. Indeed, best execution policies require that trading desks justify their choice of execution strategies based on tangible metrics.

The cost of execution is directly linked to the market impact. We will thus compare the relative prices of trading at the Close or during the continuous phase. In the following we will consider only the gross prices as, at Euronext, more than 95% of Euronext turnover is charged the same trading fees whether trading at the Close or continuously. Thus, the level of execution fees does not contribute to the decision to trade continuously or at the Close on Euronext. Therefore we will make our comparison without taking fees into account.

THE JUMP ON CLOSE REPRESENTS THE COST OF TRADING AT THE CLOSE

Measuring the cost of trading at the Close

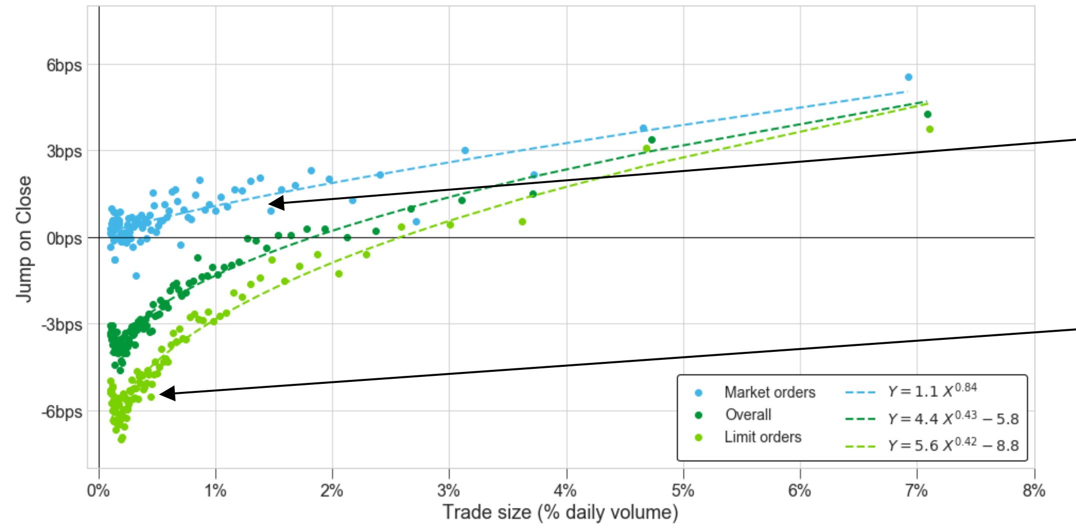
Most market impact models consider the Arrival Price as the reference price. When trading at the Close, the last continuous mid price is a natural choice of reference price. In this case, the cost of trading at the close can be expressed simply as the Jump on Close already defined earlier:

$$\text{Cost of Trading on Close} = \text{Jump on Close} = side \times \frac{P_{\text{Close}} - P_{\text{Last Mid}}}{P_{\text{Last Mid}}}$$

Jump on Close for different types of orders and the selection bias

Considering our filtered 'Trades' dataset of more than 750,000 trades (see Table 2), in Figure 24 we plot the Jump on Close with different trade sizes for Market orders (light blue), Limit orders (light green) and all orders together (dark green).

Figure 24: Jump on Close in bps for Market, Limit and all types of orders



Each dot represents the y-axis average computed on a percentile of the x-axis distribution.
Dataset Trades (filtered)
Source: Euronext

Market orders provide an upper bound for all types of orders on Close

The negative values of the Jump on Close for small sized Limit orders result from a selection bias for Limit orders

The negative values of the Jump on Close for small sized Limit orders (light green) might be disconcerting at first sight for an unfamiliar reader but this surprising observation results from a 'selection bias'. This corresponds to the fact that, when considering Buy orders for instance, it is likely that more Buy Limit orders will trade when a negative Jump on Close occurs. This is due to the fact that only a dropping closing price will trigger opportunistic Buy Limit orders with limit prices initially set below the last continuous mid price.

Market orders provide an upper bound for both Limit and Market orders on Close

For this reason, we focus on Market orders in our market impact analysis. As all Market orders give way to an execution, this means that, when observing their executions, no selection bias will occur. A second reason for this choice is that Market orders have more impact than Limit orders. As a consequence, considering the market impact of Market orders provides us with an upper bound for the market impact of a composite order, made of both Market and Limit orders.

ASSESSING THE COST OF TRADING ON THE CONTINUOUS MARKET

When trading on the continuous market, investors send their orders together with a set of specific instructions to their brokers. These instructions characterise the type of trading algorithm required as well as the choice of inputs for these algorithms. Thus, a given order can be executed in many different ways. Nevertheless, despite the variety of trading algorithms available, one can infer an average trading cost by averaging the cost of trading over a large sample of trades. To do this, one first needs to define the cost of trading.

From market impact models to the cost of continuous trading

The cost of trading during the continuous phase is measured by market impact models. They consider the Arrival Price as the benchmark. Consistently, for a parent order (defined below), the market impact is most frequently defined as follows:

$$\text{Market Impact} = side \times \frac{P_{\text{Last trade}} - P_{\text{Arrival}}}{P_{\text{Arrival}}}$$

A 'parent order' represents the initial order of Q shares (sent by an investor to a broker) split into N elementary orders (sent by the broker's trading algorithms to exchanges) each accounting for q_i shares, with P_i representing the execution price of the i^{th} elementary order. Then, to assess the cost of trading of this parent order, one must consider the average price of the execution versus the parent order's Arrival Price. Therefore, we can write:

$$\text{Cost of Continuous Trading (bps)} = side \times \frac{\sum_{i=1}^{N} q_i \times (P_i - P_{\text{Arrival}})}{Q \times P_{\text{Arrival}}}$$

Now in order to fully assess the cost of trading continuously, we need to decide which continuous market impact models can be used as benchmarks.

Reference continuous market impact models

As proprietary market impact models are not publicly available, it is necessary to rely on academic papers to assess the market impact of continuous trading. In the academic literature, market impact models are often expressed as a fraction of the daily volatility. Most models use the same inputs, as displayed in Table 3.

Table 3: Market impact models inputs

Parameter	Definition
Q	Size of the trade (in number of shares)
Q_{day}	Consolidated daily volume (across all venues) on the considered stock (in number of shares)
$q_{\%\,day}$	$100 \times \frac{Q}{Q_{day}}$ (in %)
σ	Annualised stock intraday volatility (in %)
T	Duration of execution of the trade (in min)
T_{day}	Duration of a trading day (510 min on Euronext)
$part.rate$	Trading algorithm participation rate (in %)

Of the most well-known academic market impact models, we will use the four models detailed in Table 4 as benchmarks to provide a range of estimates regarding the cost of continuous trading. Based on each market impact model, one can compute its associated cost of trading (using a simple integral). For additional details about the methodology please refer to Table 5 p47 in the Appendix.

Table 4: Reference market impact models for continuous trading

Model	Type	Universe	Period	Source
Besson and Lasnier (2020)	bps	European equity Market	2013 -2019	European Broker
Tóth et al. (2011)	% volatility	Futures Market	2007 -2010	Capital Fund Management
Bucci et al. (2019)	% volatility	US equity Market	2007 -2010	ANcerno
Bershova and Rakhlin (2013)	% volatility	US equity Market	2009 -2011	Alliance Bernstein's

For more details see Table 5 p47 and p48 in Appendix.

COMPARING THE COST OF CONTINUOUS TRADING TO THE COST OF TRADING ON CLOSE

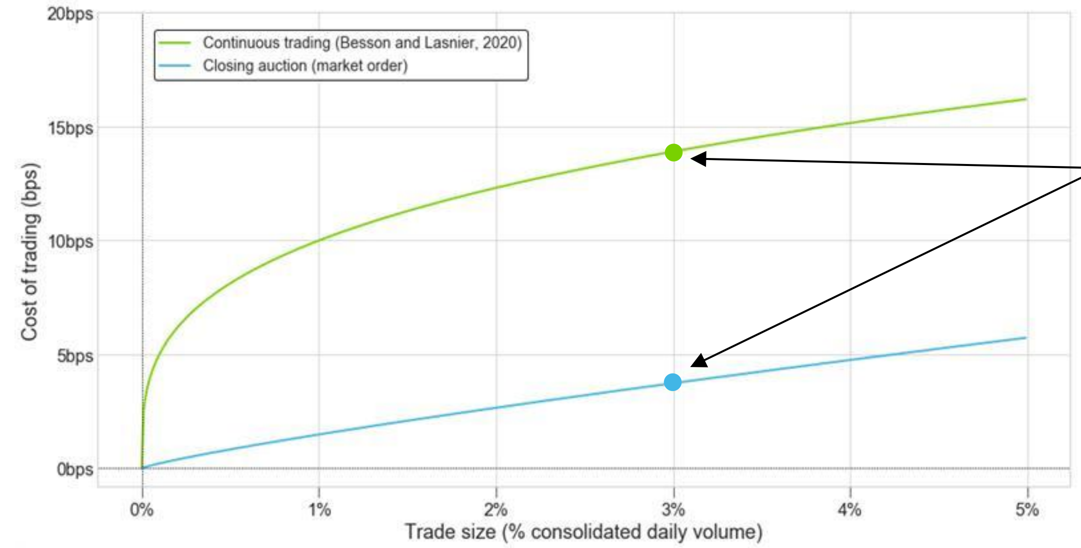
Computing the Cost of Continuous Trading for each of the four academic sources above, we then compare these results to the empirical Cost of Trading on Close measured by the Jump on Close metric. Since academic models measure the Cost of Continuous Trading in bps or in % of the daily volatility, we will consider the two cases consecutively.

Comparing the Cost of Trading in bps

For a continuous market impact reference where costs of trading are estimated in bps, and given the lack of available academic models expressed in bps without considering the intraday volatility, we rely on a working paper based on 2013-2019 European trades (Besson & Lasnier, 2020) as displayed in Figure 25. For more details about the models used, see Table 5 p47 in the Appendix.

In Figure 25 we observe the concave market impact functions for continuous trading in light green. We clearly exhibit that the continuous Cost of Trading (in green) is at least three times larger than the Cost of Trading at the Close (in light blue). As an example, the Cost of Trading for a trade representing 3% of Daily Volume amounts to 3.7 bps when trading on Close, versus 13.9 bps when trading continuously.

Figure 25: Cost of trading on Close versus Continuous in bps



We rescale the trade size expressed in % of the daily primary volume into the consolidated volume used in academic models (see Appendix p48).

Source: Euronext

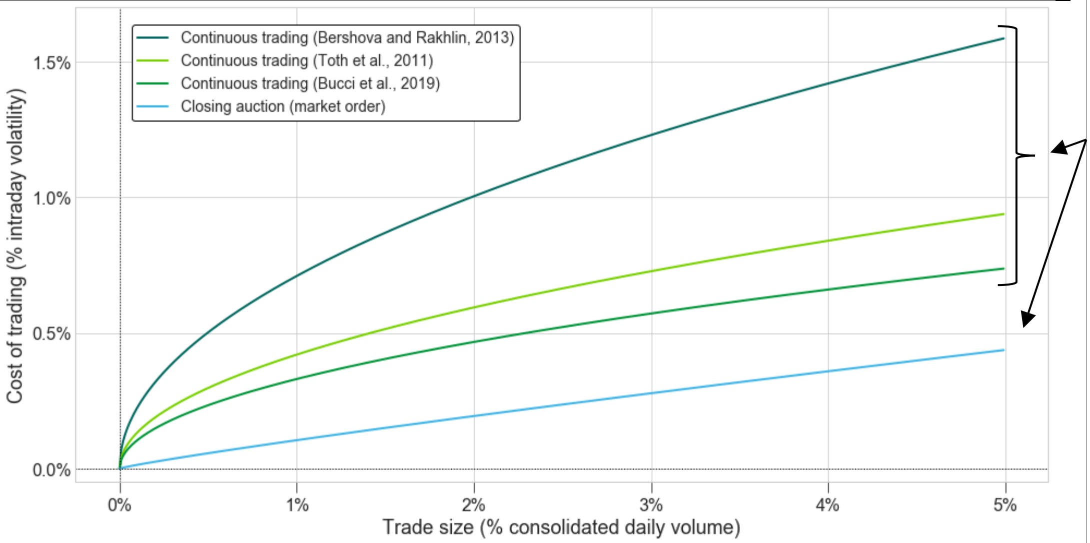
For a trade representing 3% of Daily Volume, the Cost of Trading on Close amounts to 3.7 bps on average, far less than the 13.9 bps seen for continuous trading

Comparing the Cost of Trading as a percentage of the daily volatility

In the academic literature, the market impact curve is often found to be proportional to the intraday volatility. In order to compare our empirical findings to the findings of other academics, we represent here the Cost of Trading on Close as a percentage of the intraday volatility.

We clearly observe in Figure 26 that the Cost of Continuous Trading is much greater than the Cost of Trading on Close (in light blue). Even if we take the model that gives the lowest Cost of Continuous Trading (Bucci, et al., 2019) (medium green line), we still conclude that trading on Close is more than two times cheaper than trading continuously. For instance, considering a Market order representing 3% of Daily Volume executed on a day when a 10% annualised intraday volatility was observed, trading on Close would incur an additional cost of barely 3 bps. However, the same order traded continuously would have a cost of almost 6 bps according to the model *Bucci et al.* and this cost can rise as far as 7 bps or even 12 bps for the other Cost of Continuous Trading models considered. For detailed computations please refer to Appendix p46.

Figure 26: Cost of Trading on Close versus Continuous in % of the daily volatility



Most well-known academic market impact models for continuous trading exceed the empirical cost of trading on Close

We rescale the trade size expressed in % of the daily primary volume into the consolidated volume used in academic models (see Appendix p48).
Source: Euronext

Enforcement of best execution favours executions on Close

This larger cost associated with continuous trading comes as no surprise to market participants as the most liquid time of the day is the closing auction. As such, it is to be expected that the Cost of Trading, which represents the cost of sourcing liquidity, is the lowest when the liquidity of the orderbook is at its highest.

As a consequence, it is normal that trading on Close, being the least costly type of trading, becomes increasingly popular. Nevertheless, intraday trading remains essential in order to benefit from intraday price opportunities, or when the fund manager's short-term alpha offsets the greater market impact of continuous trading.

INTERNALISATION AND VOLATILITY OF CLOSING AUCTIONS

In this final part of our paper, we investigate the relationship between the characteristics of the closing auction orderbook and the resulting Jump on Close.

We study this question because closing auction mechanisms are being challenged by new practices from market participants who are increasingly inclined to internalise orders; instead of submitting all orders to closing auctions, they tend to submit only the aggregated netted volume of orders to exchanges' closing auctions. In particular we want to address two main questions:

- What is the main characteristic of the orderbook that drives the Jump on Close?
- Is order internalisation detrimental to the volatility of closing auctions?

THREE KEY CHARACTERISTICS OF CLOSING AUCTIONS ORDERBOOKS

Since almost all orders are either Limit or Market orders, a first approach to characterise closing auctions is to examine the split across Market and Limit orders for both Buy and Sell orders as represented in Figure 27.

Only three different types of matches occur: two Market orders, a Market and a Limit order, or two Limit orders. Hence three main metrics are particularly useful:

- The 'Market Order Imbalance' characterises the imbalance between Buys and Sells among Market orders. A positive imbalance means that there is a larger volume of Buy Market orders over Sells.

$$\text{Market Order Imbalance} = 100 \times \frac{Q_{\text{Market Buy}} - Q_{\text{Market Sell}}}{Q_{\text{Close}}}$$

- The 'Market Order Buffer' characterises the proportion of auction trades made of two Market orders. This type of match is the most likely to decrease as order internalisation becomes more frequent. Indeed, Market orders are operationally simpler to net rather than being sent to a closing auction.

$$\text{Market Order Buffer} = 100 \times \frac{min\left(Q_{\text{Market Buy}}, Q_{\text{Market Sell}}\right)}{Q_{\text{Close}}}$$

- The 'Limit Order Buffer' characterises the proportion of auction trades made of two Limit orders. For obvious reasons, the sum of the absolute Market Order Imbalance, the Market Order Buffer and the Limit Order Buffer totals 100%.

$$\text{Limit Order Buffer} = 100 \times \frac{min\left(Q_{\text{Limit Buy}}, Q_{\text{Limit Sell}}\right)}{Q_{\text{Close}}}$$

Where Q stands for the volume executed on Close that we split upon order types and sides.

Figure 27: Structure of the closing volume by order type and side

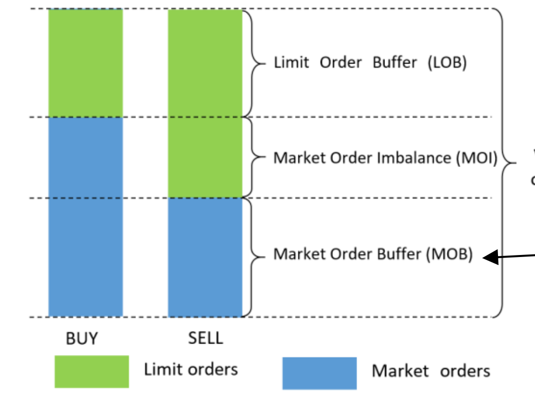
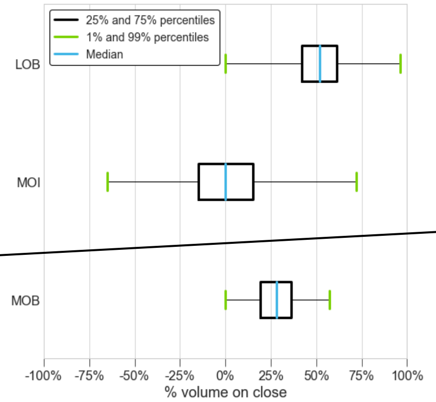


Figure 28: Buffers and Imbalance distributions



Dataset Daily (filtered)
Source: Euronext

Market Order Buffers are the most prone to internalisation, subsequently preventing some Market orders from being sent to exchanges

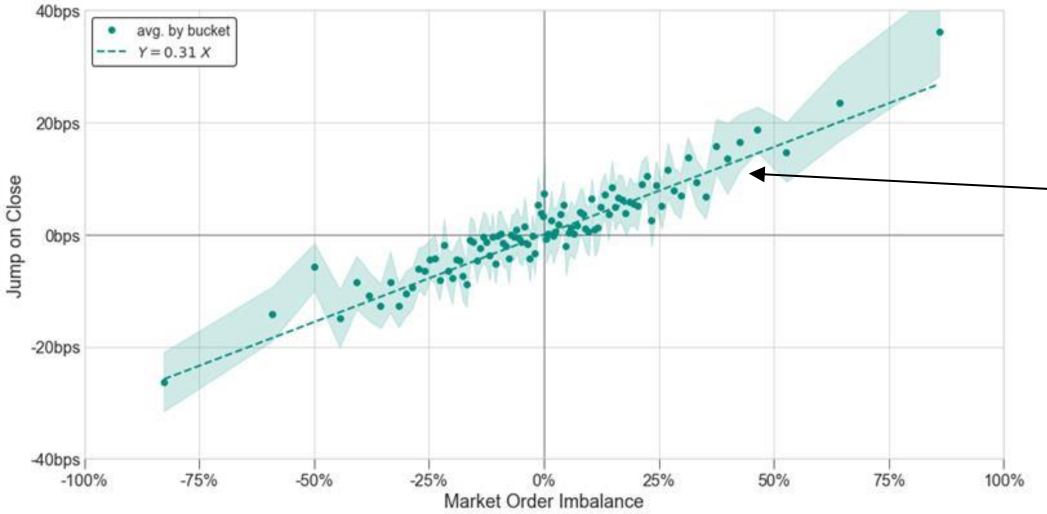
- Empirically, the Market Order Buffer (MOB) amounts to 30% in median as displayed in Figure 28.
- The absolute Market Order Imbalance (MOI) amounts to 20% in median but its quartiles range from −15 to +15% in signed value.
- Finally, the Limit Order Buffer (LOB) represents slightly more than 50% in median.

THE MARKET ORDER IMBALANCE DRIVES THE JUMP ON CLOSE

Market orders are given priority over Limit orders in the matching engine at the Close. Therefore, it is expected that they will have a key role in fixing the closing price. To conduct this analysis more precisely we will study the relationship between the Jump on Close and the Market Order Imbalance.

More precisely, in Figure 29 we plot for each percentile of Market Order Imbalance the corresponding average Jump on Close for nearly 10,000 auctions in our ‘Daily Dataset’. We observe an almost linear relationship with an R^2 of 11% on the raw underlying distribution. This clearly shows the key role played by the Market Order Imbalance in fixing the Jump on Close.

Figure 29: Jump on Close versus Market Order Imbalance



Each dot represents the y-axis average computed on a percentile of the x-axis distribution.
Dataset Daily (filtered)
Source: Euronext

The Market Order Imbalance drives the Jump on Close

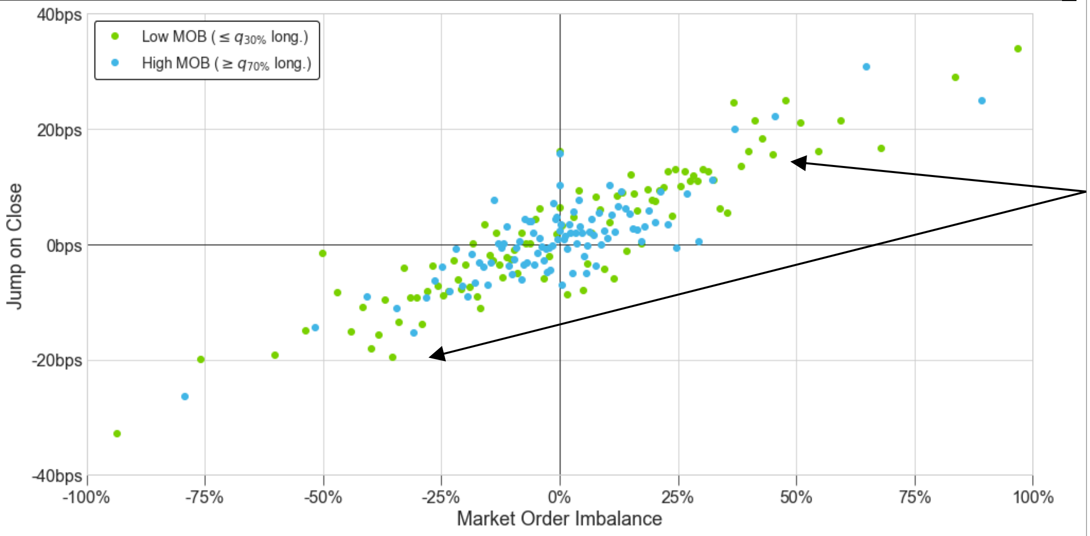
SMALLER MARKET ORDER BUFFER TRANSLATES INTO MORE AUCTION VOLATILITY

Distributions of Jump on Close for large and small Market Order Buffers

Another interesting type of matched orders to study is the Market Order Buffer. Indeed, this type of orders supposedly does not take part in the price formation process, as Market orders of opposite sides are paired without really interacting with the limit orderbook. Nonetheless, our observations show that they play a role in price efficiency as they still contain information about the liquidity available.

In order to study specifically the effects of the Market Order Buffer on the Jump on Close, for each stock we have considered the 30% days with the largest Market Order Buffer ('$\geq q_{70\%}$ long.'), and the 30% days with the smallest Market Order Buffer ('$\leq q_{30\%}$ long.'). In Figure 30 we observe that the smallest Market Order Buffer (green dots) shows on average a larger dispersion of Jump on Close than the largest Market Order Buffer (blue dots).

Figure 30: Jump on Close versus Market Order Imbalance for different Market Order Buffers



We observe that the smallest Market Order Buffer instances (green dots) show a larger dispersion of Jump on Close than the largest Market Order Buffers (blue dots)

Each dot represents the y-axis average computed on a percentile of the x-axis distribution.
Dataset Daily (filtered)
Source: Euronext

The standard deviation of the Jump on Close and the Market Order Buffer

This observation regarding the standard deviation of the Jump on Close is confirmed in Figure 31 where we observe that the smallest Market Order Buffer distribution (green dots) seems to be above the distribution of the largest Market Order Buffers (blue dots). This intuition is confirmed by Figure 32, which displays the decreasing standard deviation of Jump on Close estimates from 27 bps to 22 bps with larger Market Order Buffers. It is also notable that even for a given Market Order Imbalance, the standard deviation of the resulting Jump on Close is larger for smaller Market Order Buffers.

Figure 31: Jump on Close standard deviation versus Market Order Imbalance

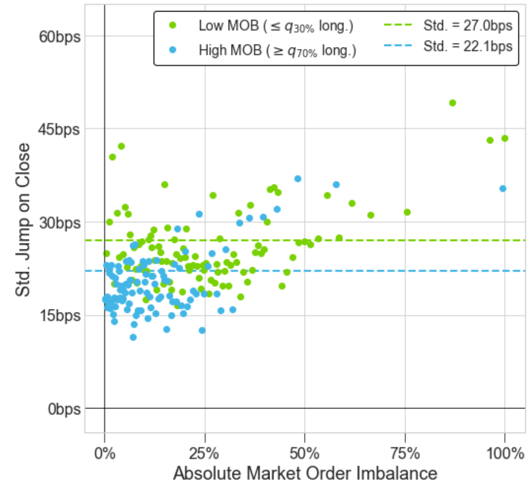
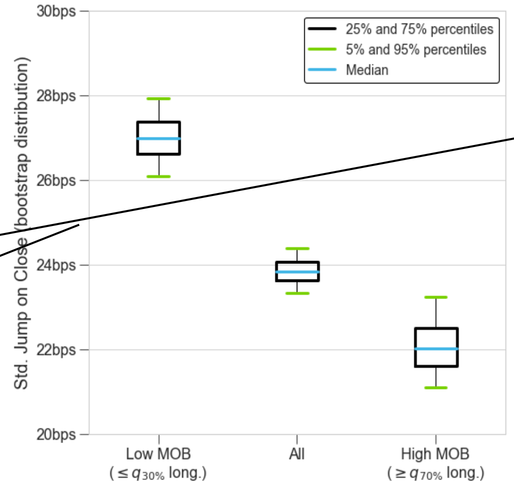


Figure 32: Jump on Close standard deviation versus Market Order Buffer, (Bootstrap distribution)



For a given Market Order Imbalance, the standard deviation of the resulting Jump on Close is larger for smaller Market Order Buffers (green)

On the left panel, each dot represents the y-axis standard deviation computed on a percentile of the x-axis distribution.
Dataset Daily (filtered)
Source: Euronext

In order to observe this phenomenon more precisely, we have also displayed for smaller and larger Market Order Buffers their corresponding Market Order Imbalance observed for such values (Figure 33 and Figure 34).

In conclusion we show that a smaller Market Order Buffer is associated with both a larger Market Order Imbalance standard deviation, from 21% to 33%, and a larger standard deviation of the Jump on Close, from 22 bps to 27 bps (Figure 34).

Figure 33: Market Order Imbalance distribution controlled by Market Order Buffer

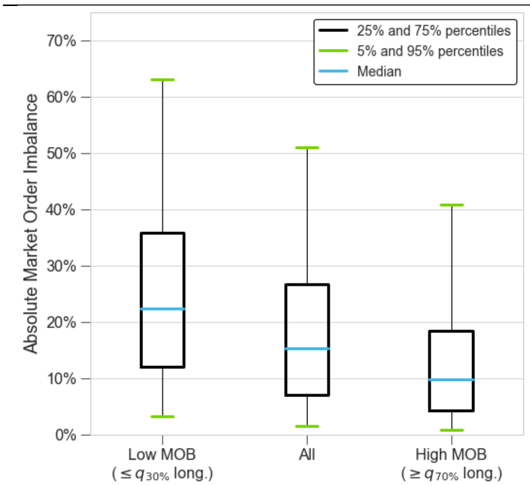
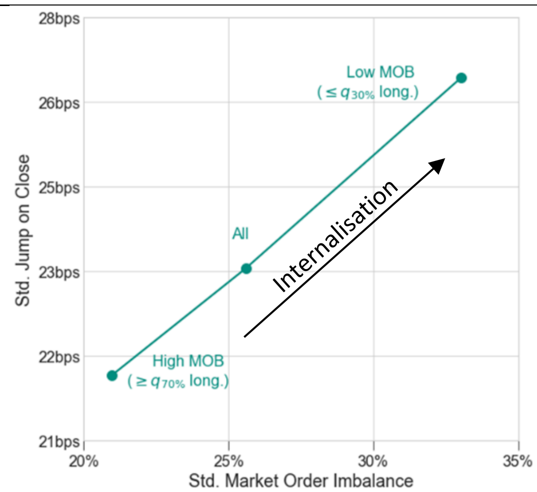


Figure 34: Jump on Close standard deviation versus Market Order Imbalance standard deviation



A smaller Market Order Buffer is associated with an increase in the standard deviations of both the Market Order Imbalance and the Jump on Close

Dataset Daily (filtered)
Source: Euronext

This shows that the Market Order Buffer acts as a stabiliser for the Market Order Imbalance, but also for the standard deviation of the Jump on Close, which measures the auction volatility.

Order internalisation and the increase in volatility of the Jump on Close

Market Order Buffers are prone to decrease due to the rising trend in Market order internalisation. Instead of sending all their orders to the closing auction, some participants are more frequently netting their orders internally, subsequently preventing some Market orders from being sent to exchanges. Therefore, we evidence the critical role of Market Order Buffers in reducing auction volatility as measured by the standard deviation of the Jump on Close.

As regulators carefully study the quality and the resiliency of the price formation around closing auctions, we draw attention to the risk that order internalisation leads to an increase in the volatility of closing auctions.

CONCLUSION

Thanks to our complete access to the orders submitted by every participant, we are able to reconstitute the full picture of market impact on Close and to evidence new facts. In each of the main parts of this paper we have established key results.

In Part 1 we have shown that Proprietary Desks and Brokers represent the largest share of the turnover on Close while Liquidity Providers and Retail Brokers account for less than 5% of the turnover (see Figure 5, p8). We have also demonstrated that closing auction limit orderbooks display characteristic asymmetric shapes (see Figure 7, p9) and that most order submissions take place at either the beginning or the end of the deterministic time span (Figure 9, p10).

In Part 2 we have established that on average the indicative volumes rise without reverting during the Call phase (Figure 10, p11), and in contrast we have evidenced the average overreaction of the estimated indicative price (Figure 13, p13). We have further shown that this overreaction was amplified with intraday volatility (Figure 15, p13) and that this reversion is explained by the imbalance of newly submitted Limit orders (Figure 16, p15).

In Part 3 we have described the market impact dynamics of a new Market order. We exhibited the Instantaneous Impact and its subsequent Decay in order to account for the Impact on Close (Figure 19 p18). Lastly we have shown that earlier order submission attenuates the Jump on Close compared to later submission (see Figure 22 and Figure 23 p20 and 21).

In Part 4 we have highlighted that the Jump on Close, which represents the cost of trading on Close, is much smaller than the cost of trading in the continuous market. This finding is corroborated by the most well-known academic market impact models based on real executions (see Figure 25 and Figure 26, p26 and 27). This strong finding accounts for the rise in the closing auction market share as best execution practices emphasise that execution should be conducted in the most cost-effective way.

Finally, in Part 5 we have shown that the Market Order Imbalance is a key driver of the Jump on Close (Figure 29, p30). Subsequently we have evidenced that a smaller Market Order Buffer is associated with more volatile closing auctions (increased standard deviation of the Jump on Close, see Figure 31, p32). This finding alerts us of the potential risk arising from an increase in market order internalisation, which is likely to reduce the Market Order Buffer.

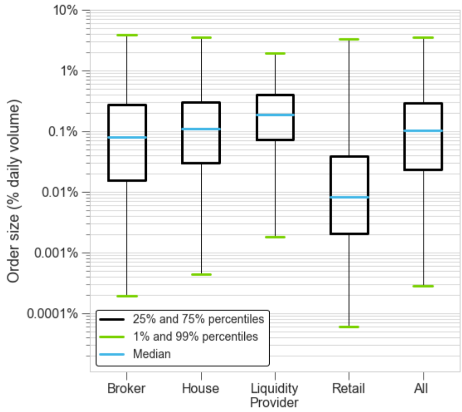
We hope that our paper will enable more public research on closing auctions so as to help investors assess independently their execution policy. We already clearly evidence the very small market impact of trades executed on Close in comparison to continuous trading. Further investigation is needed to specifically assess the market impact of Limit orders as a function of their limit price. We also hope to contribute to the growing debate on the resilience of closing auctions and the internalisation of orders.

APPENDIX

SUPPLEMENTARY ANALYSIS

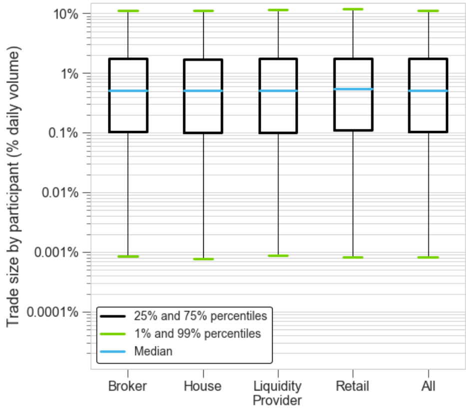
Order size, Indicative volume and price profiles

Figure 35: Order sizes on Close



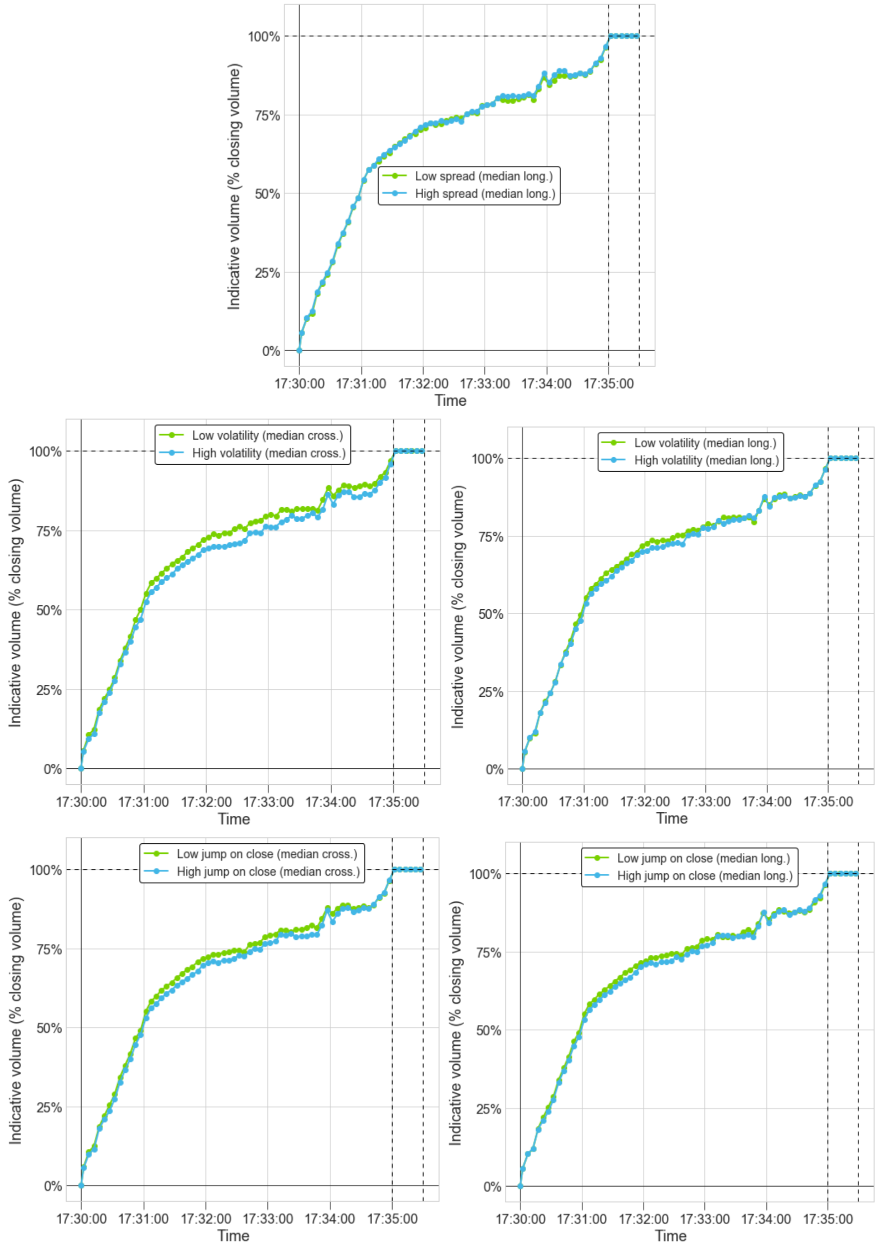
Dataset Last Modif. (unfiltered)
Source: Euronext

Figure 36: Trades on Close aggregated by participants



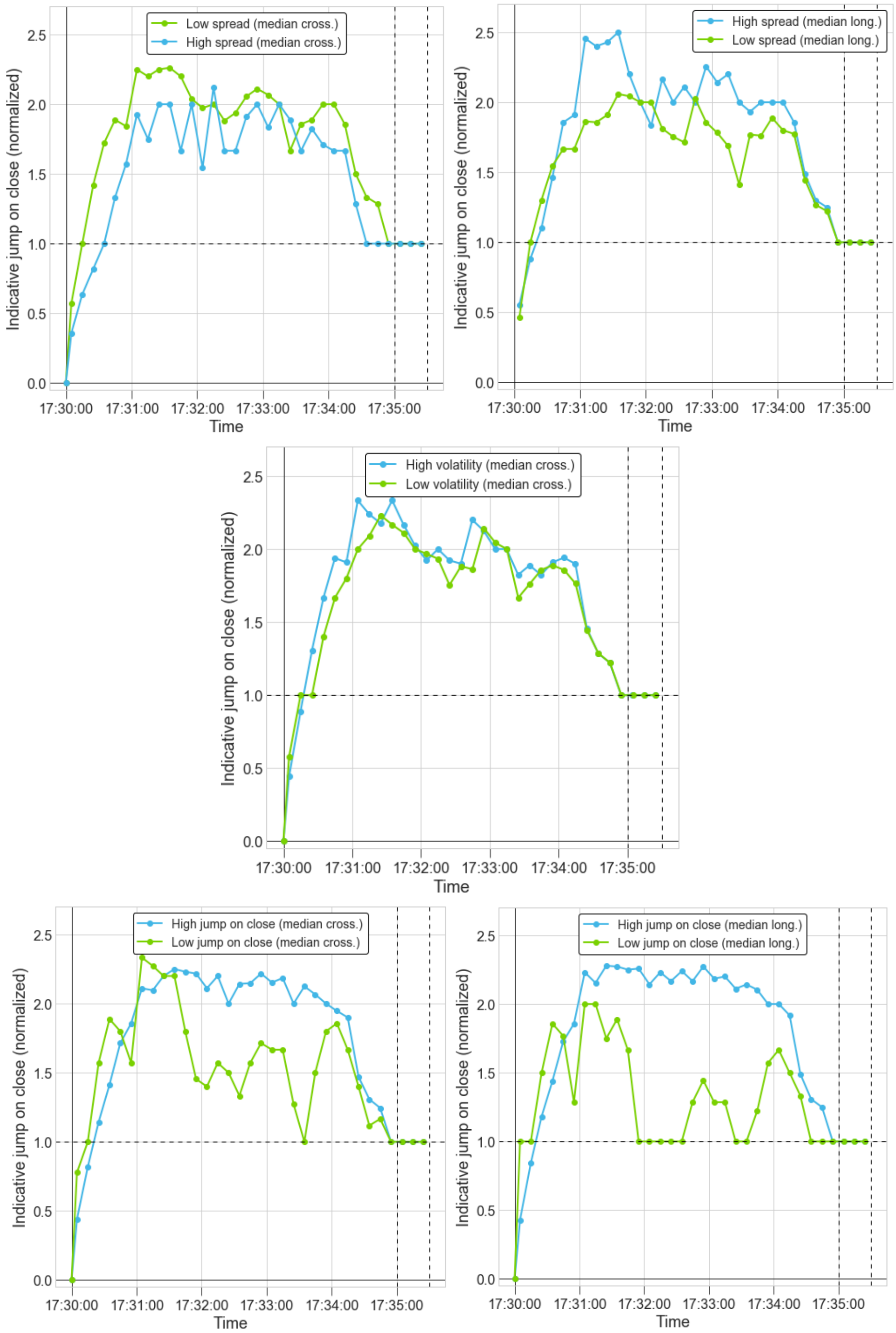
Dataset Trades (filtered)
Source: Euronext

Figure 37: Indicative volume formation profile conditioned by different parameters



Each dot represents the y-axis average computed for each 5 seconds interval of the Call phase.
Dataset Updates (filtered)
Source: Euronext

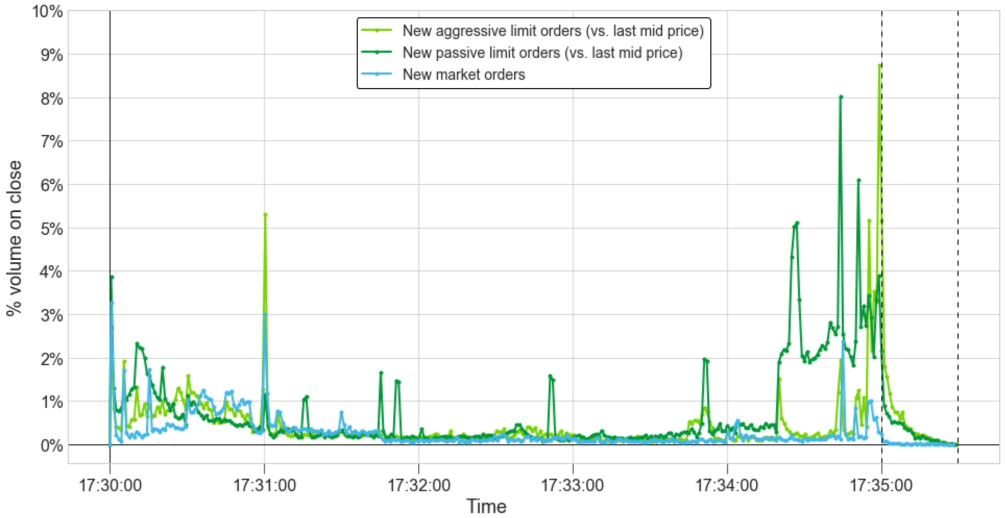
Figure 38: Indicative price formation profile conditioned by different parameters



Each dot represents the y-axis average computed on a percentile of the x-axis distribution.
Dataset Updates (filtered)
Source: Euronext

Limit order submission during the Call phase

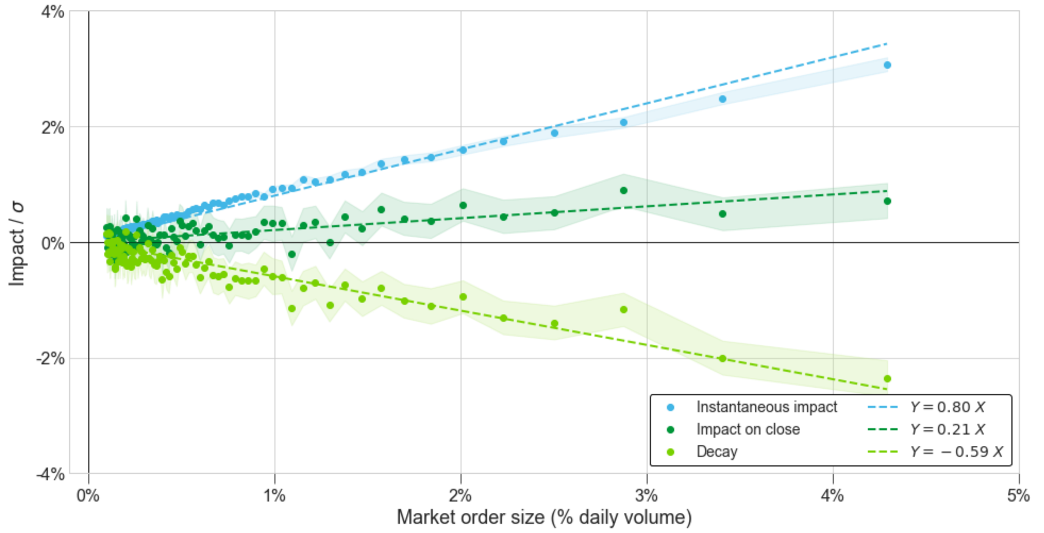
Figure 39: Order submission during the Call phase: price makers are more present at the end of the Call phase



Limit orders are split between 'aggressive' and 'passive' according to their limit price versus the last mid price.
Each dot represents the y-axis average computed for each second of the Call phase.
Dataset Updates (filtered)
Filter: New orders only (4,622,332 orders)
Source: Euronext

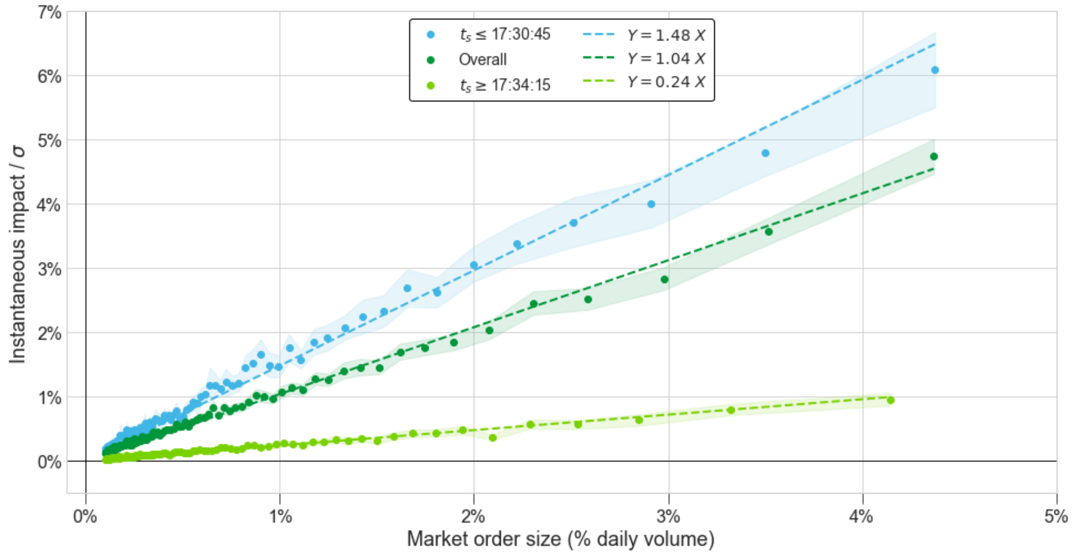
Instantaneous market impact and Decay expressed in percentage of the intraday volatility

Figure 40: Decomposition of the Market Impact on Close in percentage of the intraday volatility



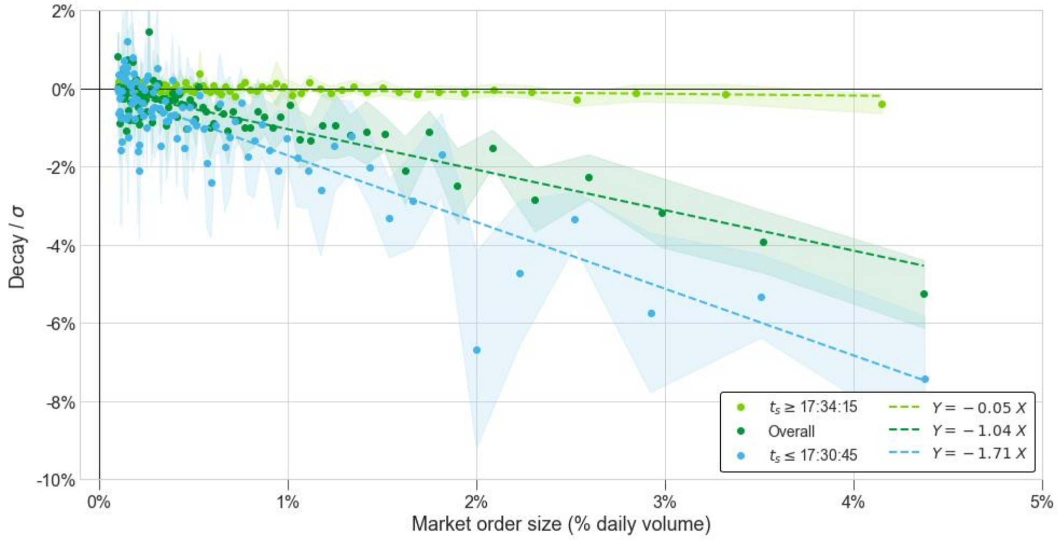
Each dot represents the y-axis average computed on a percentile of the x-axis distribution.
Dataset Last Modif. (filtered)
Filter: $|\text{instant. impact}| \leq q_{99\%} = 128\text{bps}$ (255,956 orders)
$+ |\text{impact on close}| \leq q_{99\%} = 534\text{bps}$ (254,028 orders),
$+ |\text{decay}| \leq q_{99\%} = 534\text{bps}$ (253,989 orders)
Source: Euronext

Figure 41: Instantaneous Impact as percentage of intraday volatility



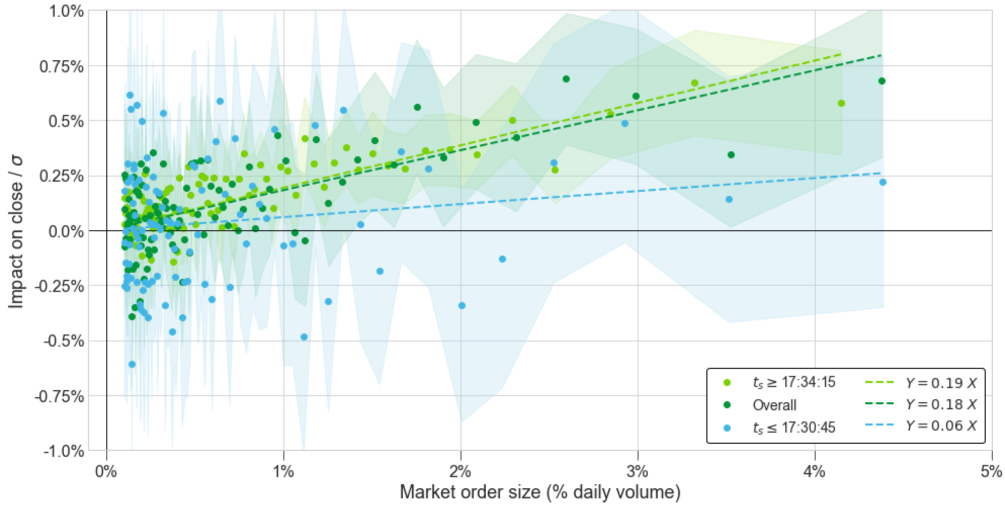
Each dot represents the y-axis average computed on a percentile of the x-axis distribution.
Dataset Last Modif. (filtered)
Filter: instant. impact $\leq q_{99.9\%} = 916\text{bps}$ (236,032 orders)
Source: Euronext

Figure 42: Decay as percentage of intraday volatility



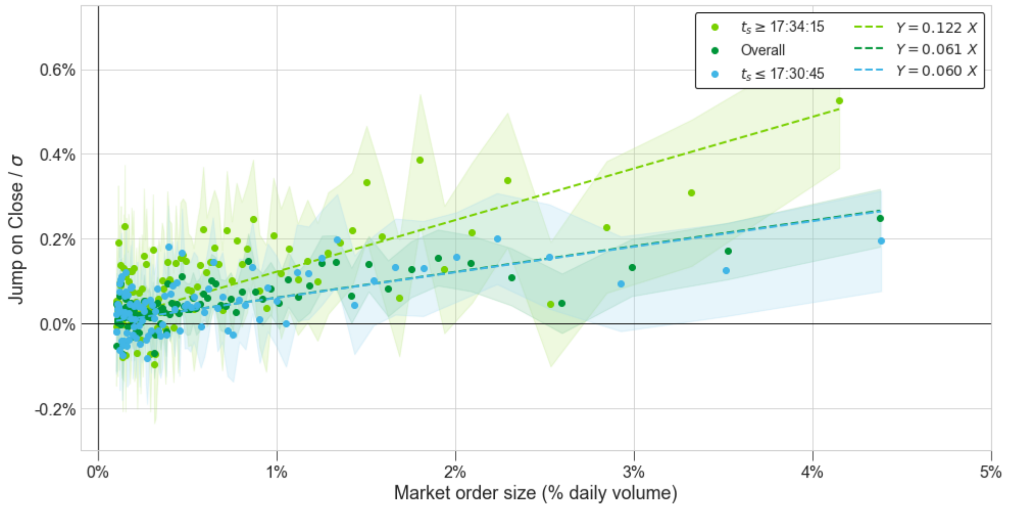
Each dot represents the y-axis average computed on a percentile of the x-axis distribution.
Dataset Last Modif. (filtered)
Filter: decay $\leq q_{99.9\%}$ = 6,544bps (236,035 orders)
Source: Euronext

Figure 43: Impact on Close as percentage of intraday volatility



Each dot represents the y-axis average computed on a percentile of the x-axis distribution.
Dataset Last Modif. (filtered)
Filter: impact on close $\leq q_{95\%}$ = 254bps (245,614 orders)
Source: Euronext

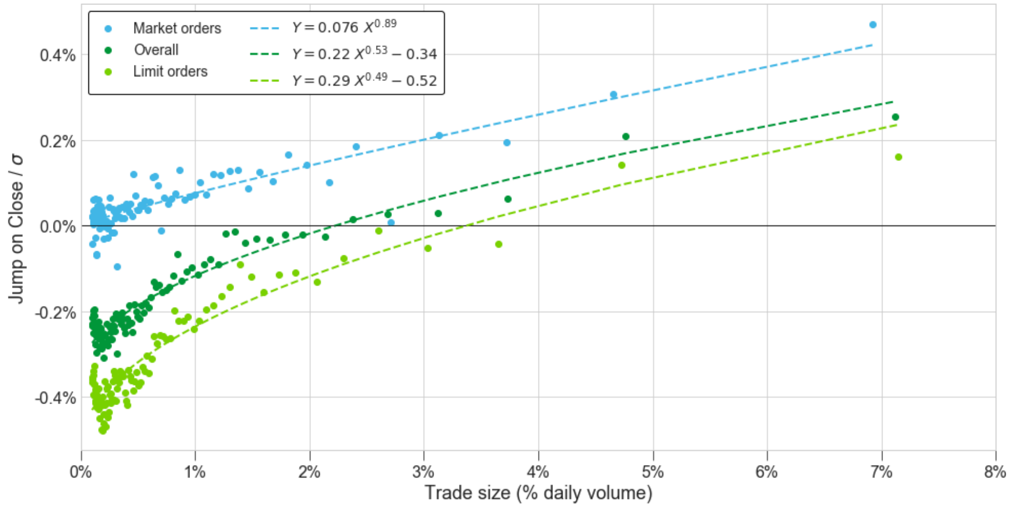
Figure 44: Jump on Close as percentage of intraday volatility by time of submission



Each dot represents the y-axis average computed on a percentile of the x-axis distribution.
Dataset Last Modif. (filtered)
Source: Euronext

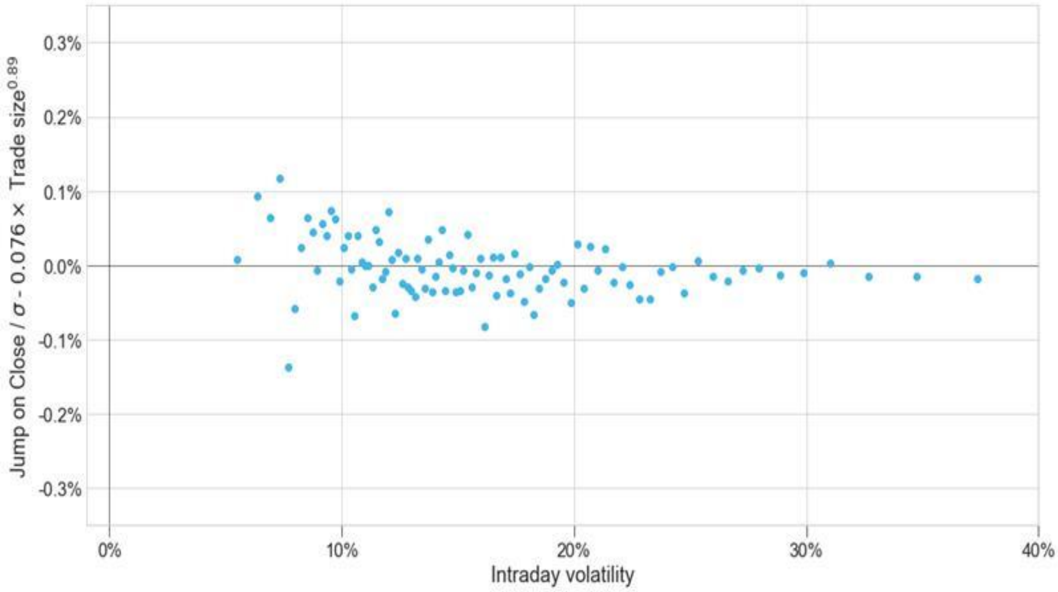
Market impact on Close as percentage of the intraday volatility

Figure 45: Jump on Close as percentage of intraday volatility



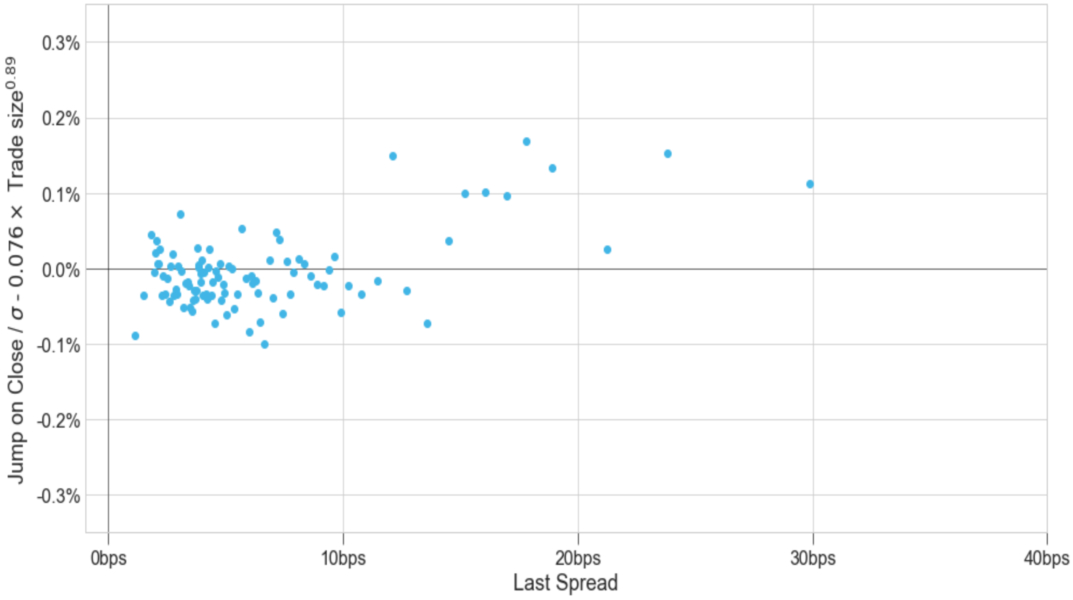
Each dot represents the y-axis average computed on a percentile of the x-axis distribution.
Dataset Trades (filtered)
Source: Euronext

Figure 46: Residuals of market impact model (in % of volatility) versus intraday volatility:



Each dot represents the y-axis average computed on a percentile of the x-axis distribution.
Dataset Trades (filtered)
Source: Euronext

Figure 47: Residuals of market impact model (in % of volatility) versus last spread

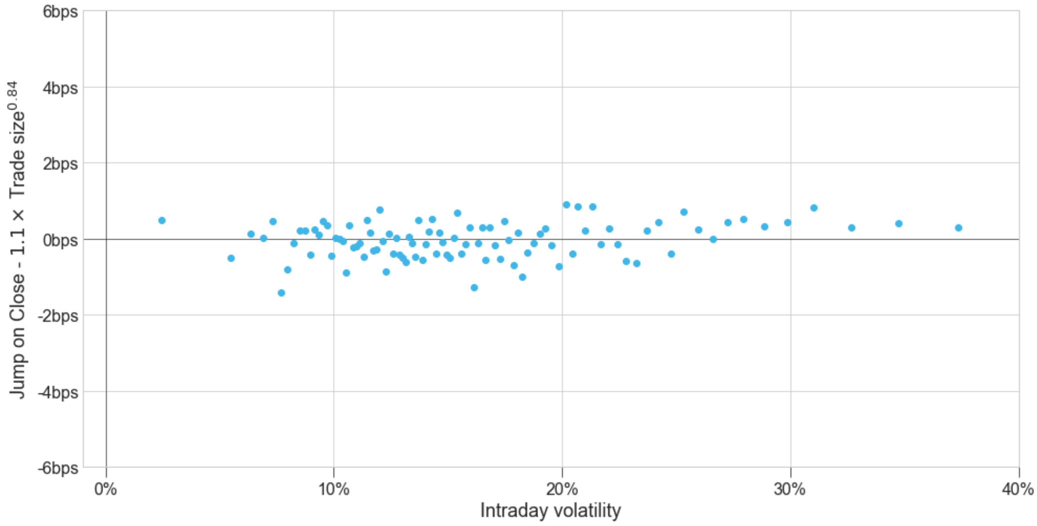


Each dot represents the y-axis average computed on a percentile of the x-axis distribution.
Dataset Trades (filtered)
Source: Euronext

Residuals of the Market Impact on Close model in bps

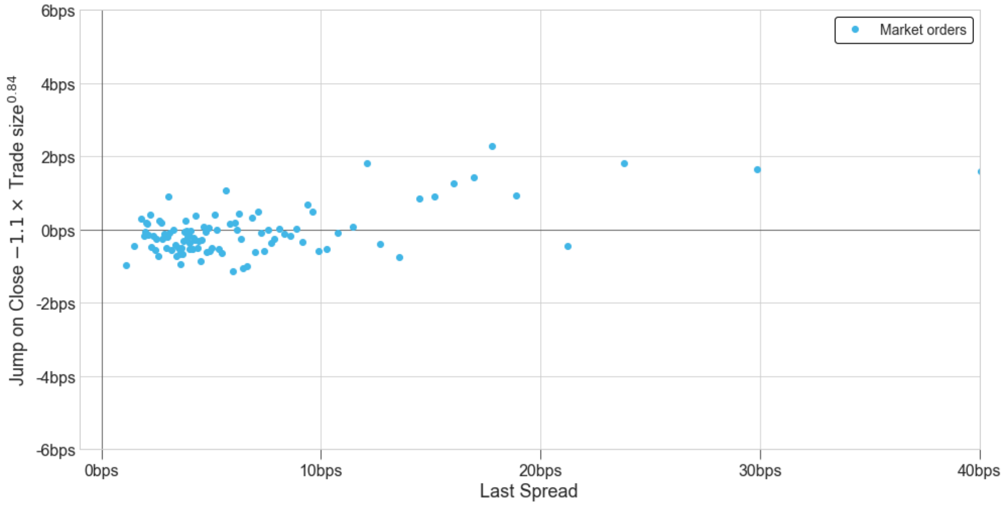
While continuous market impact models are often expressed as a percentage of the intraday volatility in order to account for the heteroscedasticity of the residuals, we show in Figure 48 and Figure 49 that the dispersion of the residuals of our plain Jump on Close model do not display any strong dependency with intraday volatility or with spreads. We even observe in Figure 46 that a model proportional to the intraday volatility creates a larger dispersion of residuals for small volatilities. Therefore the intraday volatility is not needed as an input for specifying our Jump on Close model.

Figure 48: Residuals of market impact model versus intraday volatility



Each dot represents the y-axis average computed on a percentile of the x-axis distribution.
Dataset Trades (filtered)
Source: Euronext

Figure 49: Residuals of market impact model versus last spread



Each dot represents the y-axis average computed on a percentile of the x-axis distribution.
Dataset Trades (filtered)
Source: Euronext

MARKET IMPACT ESTIMATE

When considering market impact, we aim to measure the price change induced by a trade. However, it is impossible to isolate the impact of one order as a price change δp_t following a trade at time t can be decomposed in two terms as follows:

$$\delta p_t = side \times I_t + \delta W_t$$

where $side \times I_t$ is the actual price change due to the trade considered and δW_t is a noise term.

We insist on the fact that I_t is not directly observable and that only δp_t is known. Nonetheless, since we define the Market Impact function as the expected price change induced by a trade, that is Market Impact $= E(I_t)$, we have:

$$\text{Market Impact} = E(side \times \delta p_t) - E(side \times \delta W_t)$$

where $E(.)$ is the expectation. Therefore, assuming that $E(side \times \delta W_t) = 0$, we can conclude that the expectation of the price change $E(side \times \delta p_t)$ is an estimate for the market impact curve.

More precisely, we will be often interested in computing an estimate for a conditional expectation throughout this paper. In the case of the Market Impact, we will want to compute an estimate for $E(side \times \delta p_t|Q)$ where Q is the size of the order considered. One can also condition with other variables such as the volatility, the spread, etc.

In a nutshell, we are in the general setting where we want to compute an estimate for $E(Y|X)$ with X and Y two random variables. If X is a discrete variable, we only have to compute $E(Y|X = x)$ for each value x taken by X. However, if X takes continuous values, one cannot use this approach. A common solution to this problem is to 'relax' the conditioning to $E(Y|X \approx x)$.

Following this paradigm, we will compute the percentiles of the X distribution and compute for each one of them the corresponding expectation for Y. We often choose percentiles so that our estimates are computed over equally sized samples. We will sometimes use instead a 'regular' sampling, where X is sampled over intervals of same width.

FROM MARKET IMPACT TO COST OF TRADING

Modelling the Cost of Continuous Trading

In order to compute the related Cost of Trading, we have to average Market Impacts along the trading trajectory of the elementary trades, whose prices can be computed using a standard market impact model. As we do not know the exact realised cost of trading (we do not know how the parent order is split into elementary orders), we take as a proxy for the Cost of Trading the average of the Market Impact curve relative to the order size. Thus, if we consider a power-law model of Market Impact (as often assumed in the academic literature) like Market Impact$(q_{\%\text{ day}}) = C \times q_{\%\text{ day}}^{\alpha}$, we have:

$$\text{Cost of Trading}(q_{\%\text{ day}}) = \frac{1}{q_{\%\text{ day}}} \int_0^{q_{\%\text{ day}}} C \times q^{\alpha}\, dq = \frac{C}{1+\alpha} q_{\%\text{ day}}^{\alpha}$$
$$= \frac{1}{1+\alpha} \times \text{Market Impact}(q_{\%\text{ day}})$$

Adjusting trade sizes normalisations

These models are functions of the order size computed as a fraction of the consolidated daily volume across all trading venues. However, throughout our study we considered daily volumes on the primary market only.

In order to compare our Jump on Close model for a given trade size (expressed in % of the primary market volume, $q_{\%\text{ primary day}}$) and the continuous cost of trading from standard market impact models (expressed in % of consolidated volume, $q_{\%\text{ day}}$), we have rescaled the volume fraction in our market impact model on Close by a factor $1/0.7$ to account for the market share of Euronext (estimated to $\approx 70\%$).

More explicitly: let us denote by f_{Close} our model of market impact on Close which takes as input the order size as a fraction of the daily volume on the primary market. Therefore, we have:

$$f_{Close}(q_{\%\text{ primary day}}) = f_{Close}\left(q_{\%\text{ day}} \times \frac{1}{0.7}\right)$$

This way, we can easily express our market impact model on Close as a function of the order size expressed in % of the consolidated daily volume across all venues.

ACADEMIC MARKET IMPACT MODEL

Table 5: Market impact models

Model	Source	Market Impact (in bps)	Cost of trading (in bps)	Universe	Period	Data
Besson and Lasnier (2020)	*Cumulative market impact of consecutive orders over one and two days: How long does the market remember past trades?* P. Besson, M. Lasnier (forthcoming Quantitative Finance 2021)	$\emptyset$	$10 \times {q_{\%\,day}}^{0.30}$	European equity Market	2013-2019	ANcerno
Tóth et al. (2011)	*Anomalous Price Impact and the Critical Nature of Liquidity in Financial Markets,* B. Tóth, Y. Lempérière, C. Deremble, J. de Lataillade, J. Kockelkoren, J.-P. Bouchaud (2011)	$0.63 \times \sigma\sqrt{q_{\%\,day}}$	$0.42 \times \sigma\sqrt{q_{\%\,day}}$	Futures Market	2007-2010	Capital Fund Management
Bucci et al. (2019)	*Slow decay of impact: insights from the ANcerno database,* F. Bucci, M. Benzaquen, F. Lillo, J.-P. Bouchaud (2019)	$\approx 0.5 \times \sigma\sqrt{q_{\%\,day}}$ (first order approximation)	$\approx 0.33 \times \sigma\sqrt{q_{\%\,day}}$ (first order approximation)	US equity Market	2007-2010	ANcerno
Bershova and Rakhlin (2013)	*The non-linear market impact of large trades: evidence from buy-side order flow,* N. Bershova, D. Rakhlin, (2013)	$0.1875 \times \sigma\sqrt{T}$	$0.71 \times \sigma\sqrt{q_{\%\,day}}$	US equity Market	2009-2011	Alliance Bernstein's

For more details see next Appendix and Table 3

DETAILS ABOUT THE ACADEMIC MODELS IMPLEMENTATION

See Table 3 p24 for more details on the underlying variables and Table 5 p47 for the references of the papers.

Besson and Lasnier model

The *Besson and Lasnier model* gives directly the Cost of Trading and takes as its only input the order size as a fraction of the consolidated daily volume. It is fitted on a database composed of 655,583 orders representing a traded amount of €450 million by a large European broker from July 2013 to December 2019. The model we use here is displayed in Figure 16 p19 of their paper (SSRN version). We used this model as its parameters were all given in the paper. So, in particular, by proving that our model predicts a lower cost of trading than the *Besson and Lasnier model*, we also prove that it predicts a lower cost of trading on Close than on the continuous phase overall using their European dataset.

$$\text{Cost of continuous trading } (q_{\%\,day}) = 10 \times {q_{\%\,day}}^{0.30}$$

Tóth et al. model

The *Tóth et al. model* gives the Market Impact and takes as its inputs the order size as a fraction of the consolidated daily volume and the intraday volatility. The model is fitted on 500,000 trades. The parameters of the model are inferred from Figure 1 p2 of their paper: they give a power-law form with exponent 0.5 for small ticks and we compute the multiplicative factor thanks to the curve displayed. We then compute the corresponding Cost of Trading as explained p46 of the Appendix.

$$\text{Market Impact}(q_{\%\,day}) = 0.63 \times \sigma\sqrt{q_{\%\,day}}$$

Bucci et al. model

The *Bucci et al. model* defines the market impact as the expected log-price change and gives a model which takes as its inputs the order size as a fraction of the consolidated daily volume and the intraday volatility. The model is fitted on nearly 8 million parent orders from the ANcerno database executed from January 2007 to June 2010 and accounting for around 5% of the total market volume on the period.

Their definition of Market Impact slightly differs from ours. Yet, we can easily show that, to a first order approximation, their definition coincides with ours as:

$$log(P_2) - log(P_1) = log\left(1 + \frac{P_2 - P_1}{P_1}\right) \approx \frac{P_2 - P_1}{P_1}$$

This approximation is moreover reasonable as the relative price change considered while addressing market impact is of order $\approx 10^{-3}$. The parameters of the model are inferred from Figure 1 p5 of their paper. Let us note that they point at two regimes present on this figure. We considered the regime of large orders ($\geq$ 0.1% DV), which is in square-root of the order size, as it is the one on which we focus on this study. We finally compute the multiplicative factor thanks to the curve displayed. We then compute the corresponding Cost of Trading as explained p46 of the Appendix.

$$\text{Market Impact}(q_{\%\,day}) = 0.5 \times \sigma\sqrt{q_{\%\,day}}$$

Bershova and Rakhlin model

The *Bershova and Rakhlin Market Impact model* takes as inputs the duration of the trade (in minutes), and the intraday volatility. Their model is fitted on 12,500 parent orders executed by Alliance Bernstein trading desk from January 2009 to June 2011. Their Market Impact model (from Figure 5 of their paper) is in square-root of the trade duration but we can express it as a function of the order size (defined as a fraction of the consolidated daily volume) by considering an average participation rate for the execution of the trade. Indeed, we have:

$$T = \frac{q_{\%\,day} \times T_{day}}{part.rate}$$

For numeric applications, we took a participation rate of 16% as it is the average for the trades reported in their paper. We then compute the corresponding Cost of Trading as explained on p46 of the Appendix, with T_{day} equal to 510 min.

$$\text{Market Impact}(q_{\%\,day}) = 0.1875 \times \sigma \sqrt{\frac{q_{\%\,day} \times T_{day}}{part.rate}} = 0.71 \times \sigma \sqrt{q_{\%\,day}}$$

REVIEW OF LITERATURE

Recent practitioners' publications on closing auctions

The structural increase in the market share of closing auctions is a global phenomenon across equity markets, and is particularly strong in European markets, as mentioned in all papers on closing auctions written by investors or regulators (AMF, 2019), (Blackrock, 2020), (SIX, 2020), (RBC Capital Markets, 2020), (Norges Bank, 2020). The cause of this global trend is likely to be attributable to various factors. One of the most popular explanations is the rise in passive investing: closing prices are valued at the Close and consequently they mostly trade on the close to minimise their tracking error. Yet it seems this is not sufficient to explain the phenomenon in its globality. The AMF (AMF, 2019), asset managers (Blackrock, 2020), (Norges Bank, 2020) and academics (Derksen, et al., 2020) propose two other additional explanations: investors may delay their executions until the close for fear of adverse selection by high-frequency market makers, which are almost not present at the Close; and best execution requirements following MiFID II. While these papers provide insightful analysis, they do not address the question of market impact on Close quantitatively. Our report aims to fill this void.

A recent study about the Swiss exchange (Frauendorfer & Müller, 2020) describes the sensitivity of closing prices to liquidity. Using simulations they show that auction prices are sensitive to a removal of a small fraction of the volume. They highlight the potential detrimental effect of order internalisation on auctions. In our paper we address this question without simulations but with statistical analysis of historical data.

Continuous market impact models

The literature on market impact is rather rich. This was one of the first topics addressed in the field of market microstructure with papers going back to 1985. The literature begins with the purely econometric linear model of Kyle (Kyle, 1985). It is now a widely acknowledged fact that the market impact curve is highly non-linear and it is believed to be a concave function of the order size (Bouchaud, 2010) as confirmed by many empirical studies, whether they were based on buy-side trade data (Bershova & Rakhlin, 2013) (Frazzini, et al., 2018) (Tóth, et al., 2011) or broker data (Bacry, et al., 2014) (Besson & Lasnier, 2020). Yet, the true functional form of the market impact curve is still debated among authors. Many advocate for the so-called square-root law (Tóth, et al., 2011) (Frazzini, et al., 2018) (Briere, et al., 2019) while others present evidence of logarithmic dependence on the size (Zarinelli, et al., 2015) or even a mixed form presenting a crossover from linear to square-root (Bucci, et al., 2018) (Bucci, et al., 2018).

Various attempts have been made to explain the concave shape of the market impact function. These are essentially three-folded. Kyle and Obizhaeva (Kyle & Obizhaeva, 2018) discussed a general form for the market impact function deduced from dimension analysis and some economic assumptions. Some authors derive a power-law form thanks to specific assumptions on the price process: Farmer et al. (Farmer, et al., 2013) propose a multi-agent model where the concavity of the market impact function stems from the distribution of large trades and the ability of market makers to detect with precision the presence of a large investor (which is questionable) while Rosenbaum and Jusselin (Jusselin & Rosenbaum, 2020) use a no-arbitrage condition to derive their market impact model. Other authors use the local shape of the orderbook and assumptions on the order flow resilience towards liquidity removal to derive their market impact model. Alfonsi et al. (Alfonsi, et al., 2010) use such method to compute optimal trading strategies depending on an orderbook shape function and a description of the order flow. Tóth et al. (Tóth, et al., 2011) introduce the idea of a latent orderbook where agents place their orders only when the reference price is close enough to their projection. This accounts for the concavity of market impact as more liquidity is revealed as the price is pushed upwards or downwards.

Another well documented phenomenon is the price relaxation following the initial market impact resulting from the completion of a trade. Many authors describe the two phases of market impact. The first phase often called ‘transient’ impact or ‘temporary’ impact corresponds to the execution phase of a trade. As soon as the trade is completed, the price reverts back to reach a level corresponding to the ‘permanent’ impact of the trade (Bershova & Rakhlin, 2013) (Bacry, et al., 2014) (Saïd, et al., 2018). This ‘decay’ or ‘relaxation’ phase is believed to last from a few hours to several days. Farmer et al. (Farmer, et al., 2013) proposed a model in which permanent impact accounts for approximately two-thirds of the temporary impact thanks to martingale assumptions on the price and fair pricing conditions. Conversely, Bouchaud (Bouchaud, 2010) and Bucci et al. (Bucci, et al., 2019) argue that there is no such thing as permanent impact on average once we take into account the underlying nature and the autocorrelation of the order flow. As presented above, the ‘market impact puzzle’ (quoting (Kyle & Obizhaeva, 2018)) is still far from being completed. Furthermore, all these studies focus on the execution of large trades through continuous trading only.

Market Impact on Close

The publicly available literature on market impact on close is surprisingly sparse and essentially boils down to (Derksen, et al., 2020) (Derksen, et al., 2020). In a theoretical paper they compute the distribution of closing price which appears to follow a normal distribution coherent with a diffusive price process. They also derive a theoretical market impact model on close for market orders, treating them as a liquidity surplus shifting the clearing price distribution. They finally evidence a concave impact curve similar to continuous models. They also link the heavy tails of the closing auction returns observed to the placement of limit orders rather than to large market orders as the impact of the latter is very often offset by new limit orders.

BIBLIOGRAPHY

Alfonsi, A., Fruth, A. & Alexander, S., 2010. Optimal execution strategies in limit order books with general shape functions. *Quantitative Finance.*

AMF, 2019. Growing importance of the closing auction in share trading volumes. *Available at https://www.amf-france.org/sites/default/files/2020-02/growing-importance-of-the-closing-auction-in-share-trading-volumes.pdf, 2019.*

Bacry, E., Iuga, A., Lasnier, M. & Lehalle, C.-A., 2014. Market impacts and the life cycle of investors orders. *Market Microstructure and Liquidity.*

Bershova, N. & Rakhlin, D., 2013. The non-linear impact of large trades: evidence from buy-side order flow. *Quantitative Finance.*

Besson, P. & Lasnier, M., 2020. Cumulative market impact of consecutive orders over one and two days: how long does the market remember past trades?. *Available at SSRN: https://ssrn.com/abstract=3529311.*

Blackrock, 2020. A global perspective on market-on-close activity. *Available at https://www.blackrock.com/corporate/literature/whitepaper/viewpoint-a-global-perspective-on-market-on-close-activity-july-2020.pdf.*

Boggousslavsky, V. & Muravyev, D., 2020. Who trades at the close? Implications for price discovery, liquidity and disagreement. *Available at SSRN: https://ssrn.com/abstract=3485840..*

Bouchaud, J.-P., 2010. Price Impact. *Encyclopedia of Quantitative Finance.*

Bouchaud, J.-P., Farmer, D. & Lillo, F., 2009. How markets slowly digest changes in supply and demand. *Handbook of Financial Markets: Dynamics and Evolution.*

Briere, M., Lehalle, C.-A., Nefedova, T. & Raboun, A., 2019. Stock market liquidity and the trading costs of asset pricing anomalies. *Available at SSRN: https://ssrn.com/abstract=3380239.*

Bucci, F., Benzaquen, M., Lillo, F. & Bouchaud, J.-P., 2018. Crossover from linear to square-root market impact. *Physical Review Letter.*

Bucci, F., Benzaquen, M., Lillo, F. & Bouchaud, J.-P., 2019. Slow decay of impact: insights from the ANcerno database. *Market Microstructure and Liquidity.*

Bucci, F. et al., 2018. Co-impact: crowding effects in institutional trading activity. *Available at https://arxiv.org/abs/1804.09565.*

Challet, D., 2018. Strategic behaviour and indicative price diffusion in Paris Stock Exchange auctions. *New Perspectives and Challenges in Econophysics and Sociophysics.*

Challet, D. & Gourianov, N., 2018. Dynamical regularities of US equities opening and closing auctions. *Market microstructure and liquidity.*

Derksen, M., Kleijn, B. & de Vilder, R., 2020. Clearing price distributions in call auctions. *Quantitative Finance.*

Derksen, M., Kleijn, B. & de Vilder, R., 2020. Effects of MiFID II on stock price formation. *Available at SSRN: https://ssrn.com/abstract=3559500 .*

Derksen, M., Kleijn, B. & de Vilder, R., 2020. Heavy tailed distributions in closing auctions. *Available at https://arxiv.org/abs/2012.10145.*

Farmer, D., Gerig, A., Lillo, F. & Waelbroeck, H., 2013. How efficiency shapes market impact. *Quantitative Finance.*

Frauendorfer, K. & Müller, L., 2020. Liquidity-related sensitivities of closing auctions in equity markets. *Available at https://www.alexandria.unisg.ch/261464/1/Iorcf_SIX_Closing_20201203.pdf.*

Frazzini, A., Israel, R. & Moskowitz, T. J., 2018. Trading costs. *Available at SSRN: https://ssrn.com/abstract=3229719.*

Garman, M. & Klass, M., 1980. On the estimation of security price volatilities from historical data. *The Journal of Business.*

Jusselin, P. & Rosenbaum, M., 2020. No-arbitrage implies power-law market impact and rough volatility. *Mathematical Finance.*

Kyle, A., 1985. Continuous auctions and insider trading. *Econometrica.*

Kyle, A. & Obizhaeva, A., 2018. The market impact puzzle. *Available at SSRN: https://ssrn.com/abstract=3124502.*

Norges Bank, 2020. The role of closing auctions in well-functioning markets. *Available at https://www.nbim.no/en/publications/asset-manager-perspectives/2020/the-role-of-closing-auctions-in-well-functioning-markets/.*

Pagano, M., Ling, P. & Schwartz, R., 2013. A call auction's impact on price formation and order routing: evidence from the NASDAQ stock market. *Journal of Financial Markets.*

Pagano, M. & Schwartz, R., 2003. A closing call's impact on market quality at Euronext Paris. *Journal of Financial Economics.*

RBC Capital Markets, 2020. *Measuring price efficiency of global closing auctions,* s.l.: s.n.

Saïd, E., Bel Hadj Ayed, A., Husson, A. & Abergel, F., 2018. Market impact: a systematic study of limit orders. *Market Microstructure and Liquidity.*

SIX, 2020. Trading InfoSnack #03: Closing Auctions: What's Hanging in the Balance?. *Available at https://www.six-group.com/dam/download/the-swiss-stock-exchange/trading/info-snacks/trading-infosnack-03.pdf.*

Tóth, B. et al., 2011. Anomalous price impact and the critical nature of liquidity in financial markets. *Quantitative Finance.*

Zarinelli, E., Treccani, M., Farmer, J. D. & Lillo, F., 2015. Beyond the square-root: evidence for logarithmic dependence of market impact on size and participation rate. *Market Microstructure and Liquidity.*

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