# Quality Formula

## To Measure Liquidity Providers' Contribution To Market Quality

## Functional Description

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# 1 Summary

This document describes the quality formula developed by Euronext.liffe to measure the contribution made by liquidity providers (LPs) to the quality of the market. The quality formula calculates a quality score for each of the LPs active in a contract*, which can then be used as a basis for ranking the LPs. This quality score is based on the following: seconds at best, average spread, quote size, presence in the market and the number of incoming order messages (IOMs)** used. Scores relating to seconds at best, average spread and quote size are adjusted by means of a product weighting in order to ensure that an in-the-money short-term product*** is assigned a higher value than an out-of-the-money long-term product.

All results are weighted so that they can be brought together in a formula to calculate a quality score for each contract for every LP. Weighting factors are used to determine the weight of each quality element. The quality score is used as the basis for ranking LPs, and enables LPs to be selected and deselected in accordance with a predefined promotion and relegation system.

* 'Contract' is a synonym for 'class'

** Each price that is entered in the system (whether in a new order, revision or cancellation) is an IOM. A two- sided quote therefore consists of two IOMs.

*** 'Product' is a synonym for 'series'.

# 2 Introduction

This document describes the mechanism for measuring the quality that LPs contribute to the market. This mechanism can be used to rank LPs. Appendix A contains a mathematical description of the formula. This introduction provides background information to explain why this mechanism is used.

## 2.1 Euronext Liquidity Provider System

Euronext.liffe developed the Euronext Liquidity Provider System (ELPS) in order to create a competitive two-sided market for all products throughout the entire trading day.

The goals of ELPS are to ensure:

- A fair and competitive market
- Screens full of tradable prices for all products
- Screens full of tradable prices throughout the entire trading day

A limited number of LPs are assigned to each contract traded according to this model. These LPs have undertaken to comply with a set of obligations. PMMs are obliged to maintain markets for all products in the contract, while CMMs are obliged to maintain markets for a subset of products. The prices submitted by LPs are subject to criteria relating to the minimum size and a maximum spread.

In this model LPs make and maintain markets and set the range within which trading can take place, ie a quote driven market. Trading always takes place at the best available price, but not necessarily at a price quoted by an LP. LPs have no privileges when it comes to trading, but can be assigned a preference percentage if they quoted the best price.

Investors in particular benefit enormously from access to prices for all products. Thanks to this system of market maker obligations, retail customers account for a relative high proportion of trades in ELPS contracts.

## 2.2 Limited number of LP roles

Euronext.liffe has imposed an upper limit on the number of LP roles available to strike a healthy balance between order flows and trades, and between costs and revenues. Supporting an unlimited number of LPs would require huge investments and not provide any guaranteed returns. The number of roles that are made available in each contract is considered to be the optimum number.

### 2.3 Fair competition

LP roles are auctioned once a year. The challenge is to create a mechanism that assigns available roles to the best companies and gives all potential LPs a fair chance to participate in the auction. As the average LP fulfils all its obligations, the contribution LPs make to the quality of the market is taken into consideration.

### 2.4 Quality contribution elements

In keeping with the objectives of ELPS, the following quality contribution elements have been selected.

- Seconds at best
- Average spread
- Quote size
- Presence in the market
- IOMs used

A score for each element at contract level is calculated and weighted for each LP. These scores result in an overall quality score for that contract for each LP.

Note: The daily calculated quality score for a contract for a given LP is set to zero if the LP's presence during the trading day is less than 50%

LPs can be ranked using this quality score, enabling Euronext.liffe to select or deselect LPs based on their quality where necessary.

# 3 Contribution to the market at product level

The contribution to the market at product level is measured on the basis of the number of seconds the LP's price is at best, the average spread used by the LP, and the LP's average quote size.

At product level, the minimum score that can be obtained for each of these quality elements is 0, and the maximum score is 1.

Only prices submitted in market making orders (MMOs) are taken into account. These prices must comply with the LP's quote obligations[*].

Far-out-of-the-money products are excluded from the calculation of quality scores for seconds at best, average spread and average quote size. A product is considered far out of the money if the BBO spread created by the LPs is a combination of a 'no bid' price and an ask price that is less than or equal to the maximum contractually permitted spread for at least 10% of the time the product is open for trading.

In the AEX contract, PMMs are assigned two trader keys (ITMs). These ITMs should not be used to maintain markets in the same products at the same time. As, however, PMMs active in the AEX contract must be able to move their products between their two trader keys, a maximum overlap period of one minute is permitted.

The product scores for seconds at best, average spread, average quote size and presence are set to zero if the ITMs of a PMM in the AEX index are active for the same product for more than 60 seconds during a trading day.

## 3.1 Seconds at best

The number of seconds during which an LP's price is the best market is calculated. For this purpose, the best market is not the actual best market but the best market created by LPs (PMMs and CMMs). The number of seconds at best will be calculated when the product is open for trading. The number of seconds at best on the buy side is calculated separately from the number of seconds at best on the sell side, after which the average is calculated. The result for seconds at best takes account of the number of seconds the LP was present in the product. The higher the number of seconds at best, the higher the result. By definition, the outcome is a number between 0 and 1.

**((Seconds at best on buy side + seconds at best on sell side) / 2) / number of seconds LP is present in product = score for seconds at best**

[*] A two-sided market (quote) is considered valid if it complies with the LP's obligations regarding size and spread. In order to be valid, the original size must be in accordance with the size obligation.

### 3.2 Average spread

LPs have to comply with a maximum spread when making markets. Depending on market circumstances, LPs can maintain a spread that is narrower than the contractual maximum spread. A narrower spread is usually considered to contribute more quality to the market. The LP's spread indicates the range within which trading can take place. Narrower spreads improve the market and are therefore rewarded.

The LP's average spread at product level during the period the product is open for trading is calculated and time weighted.

**$\sum$ (spread * seconds) / $\sum$ seconds = average spread**

The LP's average spread is now compared with the average BBO spread for all LPs. This spread is calculated in the same way as the average spread for individual LPs.

Average spread quality points can be scored if the LP's average spread is less than twice the average BBO spread for all LPs. Normally this spread is smaller than or at the very most equal to the LP's spread.

If the outcome is negative, the score is zero, whereas an outcome higher than 1 results in a score of 1.

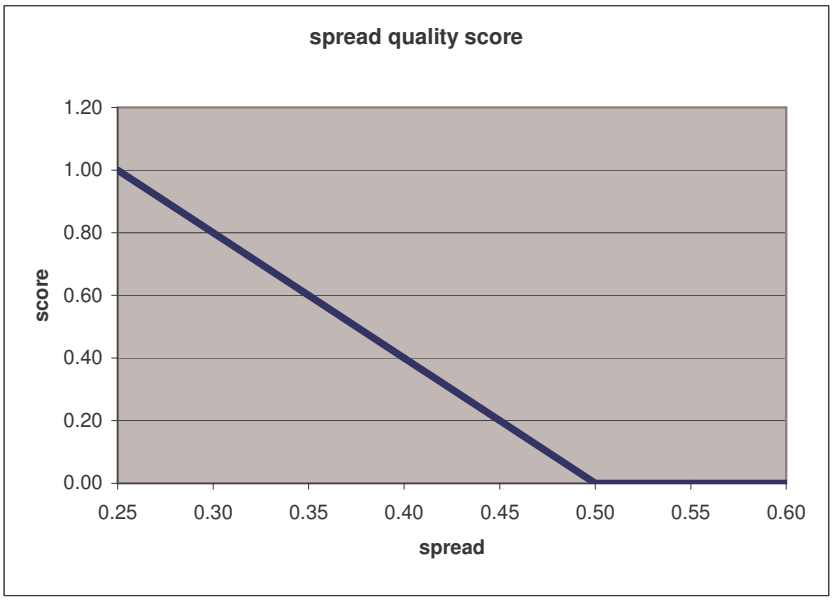
The formula is as follows.

**2 - average spread LP /average spread BBO = average spread score**

The narrower the LP's spread, the higher the outcome.

NB The average spread is calculated using valid two-sided markets (quotes).

*Example*
If a product's average BBO spread for all LPs is 0.25, an LP scores quality points when its average spread is 0.50 or less.




### 3.3 Quote size

LPs are obliged to enter quotes that comply with the contractual minimum quote size requirement. As quotes for a higher size add liquidity to the market, the additional quote size is rewarded up to a predefined level, provided the quote also complies with the contractual spread obligation.

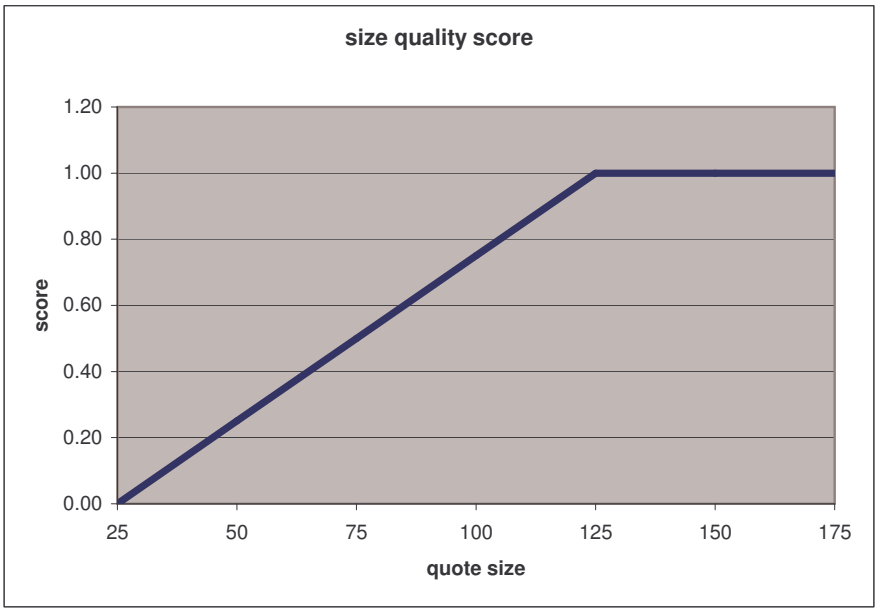
The average quote size is time weighted and calculated on the basis of the lower of the two sides.

**∑ (size of the lower side * seconds) / ∑ seconds = average size**

**(average size / contractual size – 1) * (1 / (S – 1)) = quote size score**

The range in which quote size quality points can be scored is S. S is set to five, and points can be scored if the average quote size is up to five times the contractual quote size. Higher average quote sizes are awarded the same score as average quote sizes of five times the contractual quote size.

*Example*
S is set to five, and the contractual quote size obligation for a particular product is 25. This means that quote sizes between 25 and 125 score points.


size quality score
1.20
1.00
0.80
0.60
0.40
0.20
0.00
score
25
50
75
100
125
150
175
quote size

# 4 Contribution to the market at contract level

Seconds at best, average spread and quote size are all measured at product level, but the overall quality contribution score has to be calculated at contract level. This score needs to reflect the LP's presence in the markets for all the products relating to that contract.

Quality contributions for at-the-money options are rated more highly than contributions relating to out-of-the-money options or in-the-money options. Furthermore, quality contributions relating to short-term options are rated more highly than those relating to long-term options. In connection with this, each product is assigned its own product weighting factor, which in turn is based on a strike factor and a lifetime factor.

## 4.1 Strike factor

The strike factor is used to calculate a value for each strike price, based on the difference between the strike price and the value of the underlying. This value is the time-weighted average price of the underlying on the day in question. The greater the difference between the time-weighted average underlying value and the strike price, the lower the strike factor. As the strike factor is based on the actual difference between this underlying value and the strike price, it is the same for both calls and puts. To reflect the complexity of pricing deep-in-the-money products compared to deep-out-the-money products, the minimum strike factor for out-of-the-money products is set at 0.1, and 0.2 in the case of at-the-money and in-the-money products.

The difference between the time-weighted average underlying value and the strike price is divided by a number that takes account of movements in the price of the underlying. This is based on a percentage of the CMM Performance Measurement (CPM) area. The CPM area is the area in which CMMs have to fulfil their obligations and is currently defined as follows.

The lower value of the CPM area is the day's low for the underlying minus 10%, whereas the upper value is the day's high for the underlying plus 10%, whereby the 10% deduction/addition may be no less than €2 and no more than €50 (provided the underlying value is traded in euros). The more the market price of the underlying moves during the day, the greater the CPM area will be, and therefore the greater the CPM factor used in the formula. The percentage of the CPM area used in calculating the strike factor has been set to 25%.

Formula for calculating strike factor:

**1 / (1 + ((strike – time weighted average underlying value) / % CPM area) ^2)**

*Example*

Contract: ABC
- Time-weighted average underlying value = 316.35
- Day's high for underlying = 316.96
- Day's low for underlying = 314.87
- Percentage of CPM area = 25%

Calculation of CPM area:

(316.96 +31.696) -(314.87-31.487) = 65.273

This produces the following strike factors.

| Strike price | Strike factor* (OTM / ATM, ITM) |
|---|---|
| < 260 | 0.10/0.20 |
| 260 | 0.10/0.20 |
| 270 | 0.11/0.20 |
| 280 | 0.17/0.20 |
| 285 | 0.21 |
| 290 | 0.28 |
| 295 | 0.37 |
| 300 | 0.50 |
| 305 | 0.67 |
| 310 | 0.87 |
| 315 | 0.99 |

| Strike price | Strike factor* (OTM/ATM, ITM) |
|---|---|
| 320 | 0.95 |
| 325 | 0.78 |
| 330 | 0.59 |
| 335 | 0.43 |
| 340 | 0.32 |
| 345 | 0.24 |
| 350 | 0.19/0.20 |
| 355 | 0.15/0.20 |
| 360 | 0.12/0.20 |
| 365 | 0.10/0.20 |
| >365 | 0.10/0.20 |

* OTM = out-of-the-money products
ATM = at-the-money products
ITM = in-the-money products

The result resembles a standard normal distribution. The larger the CPM area, the more widely the strike prices are distributed.

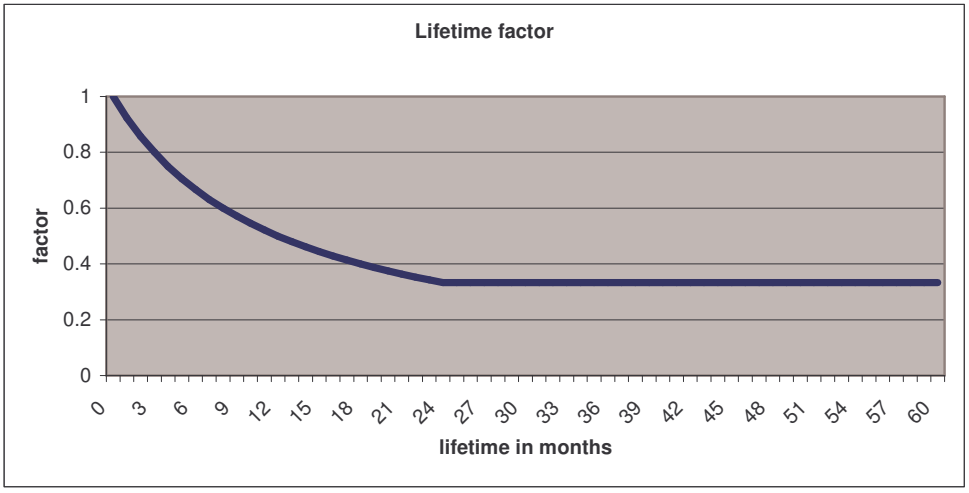
### 4.2 Lifetime factor

The lifetime factor is defined in order to enable a distinction to be made between the various remaining lifetimes of products with the same strike price. The difference in weightings between short-term and long-term options is kept to manageable levels by assigning products with a remaining lifetime of 2 years or more a fixed lifetime factor of 0.33.

The formula used to calculate the lifetime factor is as follows.

**1 / (1 + MIN(remaining life time in years, 2)) = lifetime factor**

The relationship between lifetimes and factors is shown in the following graph.




The remaining lifetimes in the graph are shown in months. However, for the purposes of the application of the formula lifetimes are expressed in terms of the remaining lifetime in years, which is calculated by dividing the number of days to expiry by 365.

### 4.3 Calculation of product weighting

The product weighting is obtained by multiplying the strike factor by the lifetime factor.

**strike factor * lifetime factor = product weight**

As the strike factor can be a minimum of 0.1 and a maximum of 1, and the lifetime factor can be a minimum of 0.333 and a maximum of 1, the maximum difference in product weighting between two products is 1:30. In the case of in-the-money options the minimum strike factor value is 0.2, which means that the maximum difference is 1:15.

### 4.4 Calculation of LP's presence

As the quality elements are measured while the LP is present in the market, it is possible that LPs that only make a market for a short period of time could still be awarded a high score for their contribution to the quality of the market. To overcome this problem, the calculation incorporates a factor at contract level that represents the LP's presence at product level, which is calculated as follows.

**number of seconds LP is present in the market / number of seconds the product is open for trading = LP presence**

### 4.5 Calculation of quality at contract level

The LP's product scores, presence per product and product weightings are used to produce a quality score at contract level. In order to give the right weighting to the LP's product scores, the scores of each product are multiplied by the LP's presence and the product weight. All product results are added together and the total is divided by the sum of product weights multiplied by LP's presence. This is repeated for the seconds at best, average spread and quote size quality elements.

In the case of seconds at best, for example, the formula is as follows.

**$\sum$ (seconds at best score * LP presence * product weight) / $\sum$ (LP presence * product weight) = seconds at best figure at contract level.**

In this way, quality scores with values of between 0 and 1 are calculated at contract level for seconds at best, average spread and quote size.

The following should be born in mind with respect to the scores produced by this formula.

- The score for each of the three elements is based only on the time the LP was present in the market.
- The total result is based on the LP's scores per product and takes account of the LP's presence and the product weightings.

NB As an LP's quality is always measured while the LP is present in the market it is possible for an LP with a low presence to achieve a high score for these three elements. This might seem unfair. However, the objective is to exclude as many other influences as possible when calculating the individual elements in order to arrive at 'pure' scores. Presence is a separate quality element, and so the LP's presence is incorporated in the quality formula itself.

Owing to its structure, the formula can be used for both PMMs and CMMs. Seconds at best, average spread and quote size are measured at product level, and combined with the LP's presence and the product weightings produce a result at contract level. It is possible to measure the quality contributed to the market by CMMs and make comparisons even if they maintain markets in different products.

This does not mean that there are no opportunities for CMMs to make an outstanding contribution. It may be easier for them to earn quality points in certain products owing to the relevant obligations and product weightings than it is in other products, and it is up to the individual CMMs to decide in which products they wish to maintain a market, bearing in mind the quality elements.

### 4.6 CMM Performance Measurement area

CMM obligations only apply in the CMM Performance Measurement (CPM) area. As a consequence, data relating to seconds at best, average spread, quote size and presence will be based exclusively on the CMM's activity within this area.

# 5 Presence

A special quality contribution element is the LP's presence in the market. Contractual obligations require an LP to be present a certain percentage of the time. For PMMs the percentage applies to the entire contract, while for CMMs the percentage applies to their obligation to be present in 10% of the products in the CPM area.

The exact way the presence is calculated is described in the document entitled *The Euronext.liffe Liquidity Provider System for trading at Euronext.liffe Amsterdam*.

In most contracts, the LP presence obligation is 90% (85% in the case of indices). If an LP's presence goes beyond its obligation, this is rewarded.

One of the key objectives of ELPS is to ensure tradable prices are available for all products throughout the entire day. For this reason, the presence calculation does not account of product weightings.

**(LP's presence– presence obligation) / (1 – presence obligation) = presence quality score**

The result is always a number between 0 and 1.

Only presence above and beyond the presence obligation is rewarded. A lower level of presence does not result in a negative score.

*Examples*
LP's presence 97%, LP's presence obligation 90% ➔ (0.97-0.9)/(1-0.9)=0.70
LP's presence 91%, LP's presence obligation 90% ➔ (0.91-0.9)/(1-0.9) =0.10
LP's presence 81%, LP's presence obligation 90% ➔ (0.81-0.9) < 0 ➔ = 0
LP's presence 97%, LP's presence obligation 85% ➔ (0.97-0.85)/(1-0.85)=0.80
LP's presence 91%, LP's presence obligation 85% ➔ (0.91-0.85)/(1-0.85)=0.40
LP's presence 81%, LP's presence obligation 85% ➔ (0.81-0.85) < 0 ➔ = 0

# 6 Usage of incoming order messages

When LPs make a market with narrower spreads or larger quote sizes than the contractual obligations stipulate and also attempt to quote the best market, they will normally use more incoming order messages (IOMs) than LPs with the same price injection model and settings but whose spreads and quote sizes are in line with the obligations. And while narrow spreads, large quote sizes and competition for the best market benefits the market, it is also necessary to avoid excessive use being made of IOMs.

With this in mind, the number of IOMs used by an LP in each contract is counted and compared with a theoretical number of IOMs for that LP. The result is a relative figure that reflects the LP's quote efficiency. Product weightings do not affect this calculation.

The following two sections explain how the theoretical number of IOMs for each LP is calculated. Section 6.3 describes how the quality score is calculated.

## 6.1 Reference model for IOM Usage

In order to be able to measure IOM usage, Euronext.liffe has developed a simple reference model to calculate a theoretical number of IOMs per product for a specific trading day. The theoretical numbers per product can be used to calculate a theoretical number of IOMs per contract. These numbers are by no means intended to instruct LPs how many IOMs they should use. They are merely objective numbers that are used to measure IOM usage by each LP.

The advantage of using this simple reference model compared with a fixed number of IOMs is that this model takes account of market conditions (underlying price movements) to a certain extent.

The first step is to calculate a theoretical opening price for each product, as follows

**Closing price + (underlying opening – underlying closing) * product delta = theoretical opening price**

As no closing price is available for new products at the time of their launch, the following rules apply in such cases.

- If at the time of the opening the market after is in line with the contractual spread, the mid-price of the market after should be used.
- If this is not possible and an opening price has been established, the opening price should be used.
- If this is also not possible, the product should not be taken into account.

Once a theoretical opening price has calculated for a product, the programme calculates a new theoretical price every time the underlying changes. This is done using the movement in the underlying and the current product delta.

**theoretical price + (new underlying – previous underlying) * product delta = new theoretical price**

- If the difference between the last theoretical price at which new IOMs were counted and the new theoretical price is more than half a price tick, two IOMs are counted and the new price is used for comparisons with new theoretical prices until new IOMs are counted.

- In the case of products with a contractual maximum spread of 0.45 or higher that have a remaining lifetime of more than three months, the difference must be more than a whole tick. This means that no new IOMs are counted for such products so long as the theoretical option price does not change by more than one price tick (up or down) compared with the previous time new IOMs were counted.

  *Example*
  A product with a remaining lifetime of six months has a theoretical price of 10.50. The contractual spread obligation is 0.80. Provided the theoretical price is equal to or greater than 10.45 and does not exceed 10.55, no new IOMs will be counted.

Once all the movements in the underlying have been processed, the number of IOM updates in all products in the relevant contract is calculated. The outcome is the theoretical number of IOMs for the contract for the current trading day.

The calculation of the number of IOMs is based on two methods. The first is based on the last price for the underlying; while the second is based on the mid point of the underlying's BBO. The method that generates the highest number of IOMs for each product is used to calculate the total theoretical number of IOMs for the relevant contract.

NB Since the objective here is to establish the number of IOMs required to respond to movements in the price of the underlying rather than anything else, this model is considered to be adequate.

### 6.2 Calculating the theoretical number of IOMs per LP

In order to arrive at a realistic calculation, the LP's presence has to be taken into account. For this reason, the calculated theoretical number of IOMs for each product for each LP is adjusted to reflect the LP's presence in a particular product.

*Example*
The theoretical number of IOMs calculated for a product is 295. LP1's presence in this particular product is 80%, while LP2's presence is 98%. For the purpose of calculating quote efficiency, 236 (295 * 0.80, rounded up) is used as the theoretical number of IOMs for LP1, while 290 (295 * 098) is used as the theoretical number of IOMs for LP2.

NB Presence is based on valid quotes. If, for example, an LP's quote is too wide in view of the LP's obligation, no presence is measured and no theoretical IOMs are counted for that period, although the actual number of IOMs used is counted in connection with the IOM efficiency figure.

The minimum theoretical number of IOMs in a product that is required to enable LPs to deal with logging on and off and delta protection breaches has been set at 10.

This calculation is repeated for each product and for each LP, which means that each LP has its own theoretical number of IOMs. The aggregate theoretical number of IOMs for all products in a contract is the LP's theoretical number of IOM's at contract level. This number is used in conjunction with the actual number of IOMs used for that contract to calculate a quality score for IOM usage.

When the theoretical number of IOMs is calculated for a CMM, this is done solely on the basis of the CMM's activity in the CPM area only. By contrast, the calculation of IOM usage is based on all IOMs, including IOMs outside the CPM area. This is done in order to discourage high IOM usage outside the CPM area.

NB All valid and invalid prices and cancellations submitted via market making orders as of 09.00 CET are taken into account.

### 6.3 Calculation of IOM quality

The number of used IOMs is calculated for each LP, as is the number of theoretical IOMs. These two numbers are then entered in the following formula to arrive at the IOM quality score.

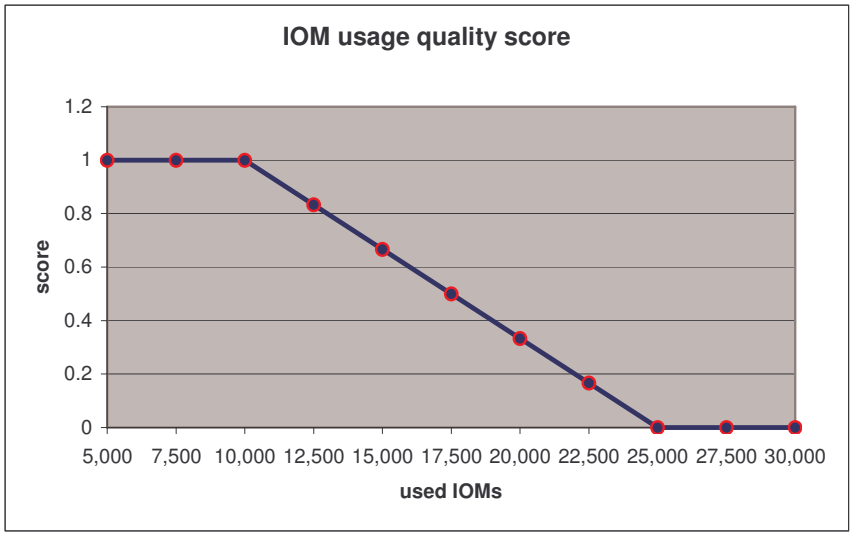
**(1 – N * (number of used IOMs / theoretical number of IOMs)) * (1 / (1 – N * α)) = quote quality score**

N sets the range in which quote quality points can be scored. If N is 0.4, quality points can be scored provided the number of used IOMs is no more than 2.5 times the theoretical number of IOMs.

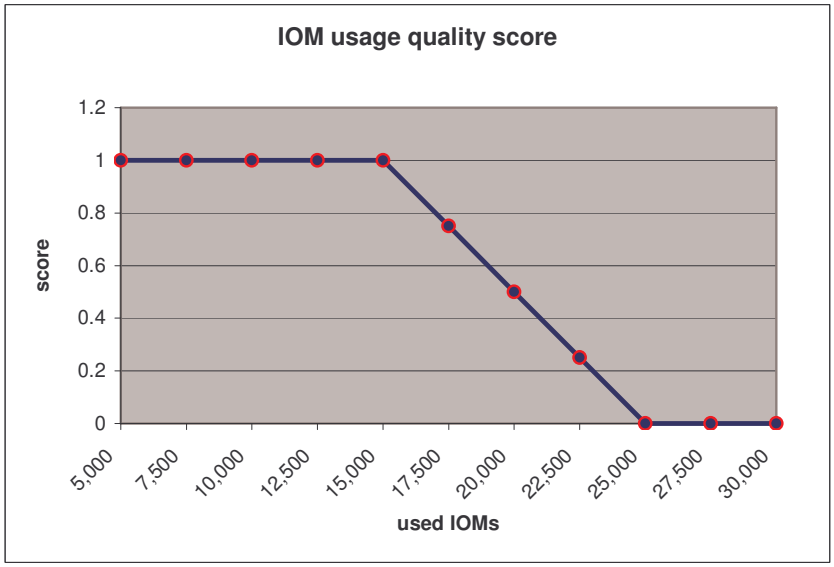
α sets the level up to which a quality score of 100% is possible. Any number in excess of this level results in a score of less than 100%.

**N** is set at 0.4 for all contracts.
**α** Is set at 1 for all equity contracts and 1.5 for all index contracts.

*Example*
The number of theoretical IOMs calculated for an LP is 10,000 for a particular product. Since N is set at 0.4 and α at 1, the potential for earning quality points is as follows:




*Example*
This second example serves to clarify the impact of α. The contract relates to an index, α is set at 1.5, and all other figures are the same as in the previous example.



When α is increased, an IOM usage quality score of 100% can still be achieved with higher usage. However, the point at which no points can be scored anymore is the same.

Again, the intention is not to impose rules for IOM usage. For this reason, Euronext.liffe considers all IOM usage up to the theoretical level to be efficient. LPs whose actual IOM usage is equal to or less than the theoretical number of IOMs are rewarded with a quality score of 100%. In the case of index products, the maximum score is still obtained if the LP's IOM usage is no more than 1.5 times the theoretical number of IOMs.

# 7 Final result

The five quality scores are used to calculate a final score for each LP in each contract. In order to make a distinction between the various quality elements weighting factors (W) have been defined.

The main objective of the quality formula is to measure the added value provided by LPs. The LPs should add value while only using a reasonable number of IOMs. The theoretical number of IOMs is considered to be sufficient, and it should be possible for all LPs to achieve the maximum score for this element. LPs that meet this requirement will be compared solely on the basis of their actual contribution to the quality of the market. In the case of LPs that do not achieve the maximum score, the reduced quote efficiency has a significant impact on their overall quality score.

The following weighting factors have been defined.

- Weighting factor for seconds at best (Wb) = 0.25 (25%)
- Weighting factor for average spread (Ws) = 0.20 (20%)
- Weighting factor for quote size (Wv) = 0.05 (5%)
- Weighting factor for presence (Wp) = 0.10 (10%)
- Weighting factor for IOM usage (Wq) = 0.40 (40%)

The following formula is used to calculate the overall quality score at contract level.

**(Qb * Wb +Qs * Ws + Qv * Wv + Qp * Wp + Qq * Wq ) = quality score**

Qb = quality score for seconds at best
Qs = quality score for average spread
Qv = quality score for quote size
Qp = quality score for presence
Qq = quality score for IOM usage

As the weighting factors add up to 1 (or 100%), the final result is always a figure between 0 and 1.

This calculation is repeated for each LP and for each contract in which it has an LP role.

## Appendix A: Quality formula

The total quality of an LP in a particular contract is a weighted sum of five quality elements. These elements are the number of seconds at the best price *($Q_B$)*, the spread in the contract *($Q_S$)*, the quote size in the contract *($Q_V$)*, the presence in that contract *($Q_P$)* and the number of IOMs used in that contract *($Q_Q$)*.

When put into a formula, the quality of LP *i* is expressed as follows.

$$Q_{tot}(i) := \frac{1}{\omega_B + \omega_S + \omega_V + \omega_P + \omega_Q} \times \left\{ \omega_B . Q_B(i) + \omega_S . Q_S(i) + \omega_V . Q_V(i) + \omega_P . Q_P(i) + \omega_Q . Q_Q(i) \right\}$$

where $\omega_B$, $\omega_S$, $\omega_V$, $\omega_P$ and $\omega_Q$ represent the weights of the five elements.

If the presence of an LP in a contract is below a threshold of 0.5, his total quality score is set to zero.

## Element 1: Seconds at best

The first quality element that an LP contributes to the market in a particular product is the number of seconds during the day when its price is the best price for that product. The LP's contribution to the quality of the market in a contract is the weighted sum of its contribution to the quality of the markets in the various products relating to that contract.

In order to measure the contribution properly, the amount of time during which the LP is active in a particular product has to be taken into account. This is indicated by the LP's presence. If $p(i,k)$ is the presence of LP $i$ in product $k$ ,then LP $i$'s presence is as follows.

$$p(i,k) := \frac{s_{market}(i,k)}{s_{open}(k)}$$

Here, $s_{market}(i,k)$ denotes the number of seconds during which LP $i$ has been making a market in product $k$ , and $s_{open}(k)$ is the number of seconds that product $k$ has been open for trading.

If $\omega(k)$ denotes the weight of a specific product, and $B(i,k)$ the quality contributed as a result of LP $i$'s number of seconds at best in product $k$, the contribution to quality made by LP $i$ as a consequence of these seconds at best ($Q_B(i)$) is expressed mathematically as follows.

$$Q_B(i) := \frac{1}{\sum_k \omega(k).p(i,k)} . \sum_k \omega(k).p(i,k).B(i,k)$$

The quality contributed as a result of LP $i$'s number of seconds at best in product $k$ ($B(i,k)$) based on the number of seconds that LP is in the market is as follows.

$$B(i,k) := \frac{(s_{bid}(i,k) + s_{ask}(i,k))/2}{s_{market}(i,k)}$$

Here $s_{ask}(i,k)$ denotes the number of seconds LP $i$ was at the best ask in product $k$, and $s_{bid}(i,k)$ the number of seconds it was at the best bid. $s_{market}(i,k)$ are defined above.

The weighting of each product depends on the relationship between its strike price and the price of the underlying (at-the-money products are given more weighting than products that are in or out of the money), and its remaining lifetime.

The following formula is used to calculate these weightings.

$$\omega(k) := T(k).L(k)$$

Here, *T(k)* denotes the strike factor and *L(k)* the lifetime factor. The strike factor is expressed as

$$T(k) := \max\left\{T_{\min}(k), \frac{1}{1+\left(\left(X(k)-X_{at}\right)/X_0\right)^2}\right\}$$

where

$$T_{\min}(k) := \begin{cases} 0.2 & k : X(k) \le X_{at} \\ 0.1 & k : X(k) > X_{at} \end{cases}$$

*X(k)* denotes the strike price of product *k*, $X_{at}$ the time-weighted mid-price of the BBO for the underlying, and $X_0$ a parameter governing the decrease of the weight *T(k)* as a function of the strike price. This parameter has been set at the width of the CMM Performance Measurement (CPM) area multiplied by parameter *w*, as follows.

$$X_0 := w.\left(1.1U_{high} - 0.9U_{low}\right)$$

Here $U_{high}$ denotes the highest price of the underlying that trading day, and $U_{low}$ the lowest price of the underlying that day. The parameter *w* is determined in such a way that *T(k)* for the next expiry month reflects the distribution of the volume traded via central market in AEX products for that expiry month. This analysis results in the following parameter value.

$$w := 0.25$$

The same parameter value is used for all expiry months and all contracts, in order to simplify matters.

The lifetime factor is expressed as follows.

$$L(k) := \frac{1}{1+\min\left\{\tau(k), \tau_{\max}\right\}/\tau_0}$$

Here, *τ(k)* indicates the remaining lifetime of product *k* in years, while the $\tau_{\max}$ and $\tau_0$ parameters are set at the following values (in years):

$$\tau_{\max} := 2$$

$$\tau_0 := 0.5$$

NB In actual calculations, the following practical considerations apply.

1. Only prices submitted by LPs in the form of market maker orders (MMOs) are taken into account.
2. Only prices relating to orders whose quote sizes comply with the LP's obligations are taken into account.
3. Products that are far out of the money, ie products without a bid price, are not taken into account when determining quality scores for seconds at best, average spread and average quote size. Technically this is achieved by setting the weights of those products to zero. A product is considered far out of the money if the BBO spread created by the LPs is a combination of a 'no bid' price and an ask price that is less than or equal to the maximum contractually permitted spread during at least 10% of the time the product is open for trading.
4. Only products within the CPM area are taken into account when calculating quality scores for CMMs for seconds at best, average spread and average quote size.

## Element 2: Average spread

The second quality element that LPs contribute to the market in a certain product relates to the average spread of its prices for that product. In line with seconds at best, the LP's average spread contribution to the quality of the market in a contract is the weighted sum of that LP's contribution in each of the products of that contract. In order to measure its contribution properly, the period of time during which an LP is active in a product has to be taken into account. If $Q_S(i)$ denotes the contribution of LP $i$ in a certain contract, $S(i,k)$ its contribution for product $k$ relating to that contract, $\omega(k)$ the weight of product $k$, and $p(i,k)$ LP $i$'s presence in product $k$, then $Q_S(i)$ can be expressed as follows.

$$Q_S(i) := \frac{1}{\sum_k \omega(k).p(i,k)} . \sum_k \omega(k).p(i,k).S(i,k)$$

The product weighting $\omega(k)$ and presence $p(i,k)$ are defined above.

The quality contributed on the basis of the average spread in a specific product is defined as follows.

$$S(i,k) := \begin{cases} 1 & \sigma(i,k) < \sigma_{bbo}(k) \\ \left(2 - \frac{\sigma(i,k)}{\sigma_{bbo}(k)}\right) & \sigma_{bbo}(k) \le \sigma(i,k) < 2\sigma_{bbo}(k) \\ 0 & \sigma(i,k) \ge 2\sigma_{bbo}(k) \end{cases}$$

Here, $\sigma_{bbo}(k)$ denotes the time-weighted average BBO spread in product $k$ as created by the LPs, and $\sigma(i,k)$ the time-weighted average spread of LP $i$ in product $k$.

## Element 3: Quote size

The third quality element that an LP contributes to the market in a particular product relates to its quote size. In line with the previous quality elements, the quote size for a contract is the weighted sum of the LP's contributions to the markets for each of the products relating to that contract, taking account of the LP's presence in each product. If $Q_V(i)$ represents LP $i$'s contribution to the market for a certain contract, $V(i,k)$ its contribution to the market in product $k$ relating to that contract, $\omega(k)$ the weighting of product $k$, and $p(i,k)$ its presence in product $k$, then $Q_V(i)$ can be expressed as follows.

$$Q_V(i) := \frac{1}{\sum_k \omega(k).p(i,k)} . \sum_k \omega(k).p(i,k).V(i,k)$$

The product weighting $\omega(k)$ and presence $p(i,k)$ are defined above.

The quality contributed on the basis of the quote size in product $k$ is defined as follows.

$$V(i,k) := \begin{cases} 1 & v(i,k) \geq N_{cap} v_{obl}(k) \\ \frac{1}{N_{cap} - 1} . \left( \frac{v(i,k)}{v_{obl}(k)} - 1 \right) & v_{obl}(k) \leq v(i,k) < N_{cap} v_{obl}(k) \\ 0 & v(i,k) < v_{obl}(k) \end{cases}$$

When calculating the time-weighted average quote size $v(i,k)$, only periods during which the quote size exceeds the contractual quote size $v_{obl}(k)$ are taken into account. The contribution is capped at a multiple of the contractual quote size for each product. This multiple $N_{cap}$ is a parameter set at the following value.

$$N_{cap} := 5$$

## Element 4: Presence

Presence in a contract beyond the contractual presence obligation is considered an additional contribution to the quality of the market. The contribution $Q_P(i)$ of LP $i$ shows a linear dependency on the LP's presence $P(i)$, starting from its obligatory presence $P_O$, as follows.

$$Q_P(i) := \begin{cases} \frac{P(i) - P_o}{1 - P_o} & P_o < P(i) \le 1 \\ 0 & 0 \le P(i) \le P_o \end{cases}$$

The LP's presence at contract level is calculated as a matter of course in order to monitor compliance with its contractual obligations. Details of this calculation are described elsewhere.

## Element 5: IOM usage

LPs need to update their quotes during the day in order to maintain a market. As long as the number of IOMs used by an LP remains within a certain range, the LP is considered to quote prices efficiently. In such cases, the LP is assigned the maximum score of 1. If a certain threshold is crossed the score is reduced, and if another threshold is crossed it does not obtain any score at all. The efficiency $Q_Q(i)$ of LP $i$ can be expressed as a function of the number of IOMs used during the day $U(i)$, as follows.

$$Q_Q(i) := \begin{cases} 1 & U(i) < \alpha.T(i) \\ \frac{1}{1 - \alpha.N}.\left(1 - N.\frac{U(i)}{T(i)}\right) & \alpha.T(i) \le U(i) < T(i) / N \\ 0 & U(i) \ge T(i) / N \end{cases}$$

The linear dependency slope is set by a dimensionless market-wide parameter $N$ divided by a contract dependant number $T(i)$, which represents the theoretical number of IOMs needed to maintain the market. This theoretical number of IOMs is calculated separately for each contract every day and takes account of movements in the price of the underlying during that day. The number is the sum of the number of IOMs for each product, $t(k)$, aggregated for all products relating to the contract, and weighted by the presence in each product.

$$T(i) := \sum_k p(i,k).t(k)$$

Here $p(i,k)$ denotes the presence of the LP, as defined above. The sum includes all products $k$ relating to the contract.

The following parameter values have been chosen following analysis of actual data.

$$\alpha := \begin{cases} 1.5 & \textit{for index option contracts} \\ 1 & \textit{for all other contracts} \end{cases}$$

$$N(i) := 0.4$$