



EQUITIES & EQUITY DERIVATIVES RISK ENGINE

Initial Margins

Methodological notes

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1 Introduction and methodology

The purpose of this document is to describe the steps for computing the *Initial Margins* of the portfolio subject to margining.

These *Initial Margins* are an enhanced, dynamic version of the current static, SPAN-like methodology.

The financial products to which the methodology described in this document is applied belong to the following categories:

- 1) Stocks;
- 2) Exchange Traded Products ETPs (ETFs/ETNs/ETCs);
- 3) Closed-end and open-end funds;
- 4) Stock *futures*;
- 5) Stock *options* (European and American);
- 6) ETF *options* (American);
- 7) Stock index *futures*;
- 8) Stock index *total return futures*;
- 9) Stock index *options* (European);
- 10) Stock *dividend futures*;
- 11) Stock index *dividend futures*;
- 12) Warrants;
- 13) Rights issues;
- 14) Certificates.

A small portion of Euronext cash legacy market bonds are cleared under the *Equities/Equity Derivatives* asset class. Their volume is negligible (annual volume in 2021 roughly equal to 0.15% of the total volume of financial instruments cleared under the asset class). Such bonds are margined according to the same methodology applied to *cash* instruments.

1.1 Position netting

Position netting is allowed between long and short positions on the same financial instrument (cash or derivative) having the same exact characteristics, including denomination currency. Position netting on the same financial instrument is allowed also if traded on different markets. In that case, risk factor data of the ‘reference’ (i.e. main) market are taken into account for margin calculation purposes.

1.2 Initial Margins calculation

While the aim of the *Mark-to-market Margins* is a backward-looking, product-specific evaluation of a Clearing Member’s portfolio of positions (comparing current conditions to

trade/previous conditions), that of the *Initial Margins* is a forward-looking (what-if) portfolio evaluation to cover potential future post-default losses.

For *Initial Margins* calculation purposes, the same (net) positions in financial instruments (with relative multipliers) which are part of a Clearing Member's portfolio and are marked-to-market are then revalued under a given set of (common, risk factor variation) scenarios.

Revaluation and margining at portfolio level allows to account for hedging, diversification and cross-correlation effects, thus introducing margin offsetting (netting) benefits.

First, the various products in the Clearing Member's portfolio are priced in the current, neutral scenario (current values of risk factors) and in the revaluation scenarios (scenario values of risk factors, which in turn are a function of current values and their returns).

Table 1: Clearing Member's portfolio's (re)valued products

Scenario	Product 1 value	Product 2 value	...	Product <i>n</i> value
Current, neutral	$P_{1, \text{current}}$	$P_{2, \text{current}}$...	$P_{n, \text{current}}$
Revaluation 1	$P_{1, 1}$	$P_{2, 1}$...	$P_{n, 1}$
Revaluation 2	$P_{1, 2}$	$P_{2, 2}$...	$P_{n, 2}$
...
...
Revaluation LP - 1	$P_{1, LP-1}$	$P_{2, LP-1}$...	$P_{n, LP-1}$
Revaluation LP	$P_{1, LP}$	$P_{2, LP}$...	$P_{n, LP}$

FX rates are necessary to convert product prices (current and revalued) in the relevant clearing currency(ies). FX rates are risk factors themselves, thus are subject to revaluation as well. Current product prices are converted employing current FX rates while revalued product prices are converted employing revalued FX rates.

Table 2: (Re)valued FX rates (*m* various product currencies for a given clearing currency *xxx*)

Scenario	FX rate 111/ <i>xxx</i>	FX rate 222/ <i>xxx</i>	...	FX rate <i>mmm</i> / <i>xxx</i>
Current, neutral	$FX_{111/xxx, \text{current}}$	$FX_{222/xxx, \text{current}}$...	$FX_{mmm/xxx, \text{current}}$
Revaluation 1	$FX_{111/xxx, 1}$	$FX_{222/xxx, 1}$...	$FX_{mmm/xxx, 1}$
Revaluation 2	$FX_{111/xxx, 2}$	$FX_{222/xxx, 2}$...	$FX_{mmm/xxx, 2}$
...
...
Revaluation LP - 1	$FX_{111/xxx, LP-1}$	$FX_{222/xxx, LP-1}$...	$FX_{mmm/xxx, LP-1}$
Revaluation LP	$FX_{111/xxx, LP}$	$FX_{222/xxx, LP}$...	$FX_{mmm/xxx, LP}$

Table 3: Clearing Member's portfolio's (re)valued products in clearing currency *xxx*

Scenario	Product 1 value in clearing currency <i>xxx</i>	Product 2 value in clearing currency <i>xxx</i>	...	Product <i>n</i> value in clearing currency <i>xxx</i>
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Current, neutral	$P_{1, \text{current}, \text{xxx}} =$ $P_{1, \text{current}} * FX_{1/\text{xxx}, \text{current}}$	$P_{2, \text{current}, \text{xxx}} =$ $P_{2, \text{current}} * FX_{2/\text{xxx}, \text{current}}$		$P_{n, \text{current}, \text{xxx}} =$ $P_{n, \text{current}} * FX_{n/\text{xxx}, \text{current}}$
Revaluation 1	$P_{1, 1, \text{xxx}} =$ $P_{1, 1} * FX_{1/\text{xxx}, 1}$	$P_{2, 1, \text{xxx}} =$ $P_{2, 1} * FX_{2/\text{xxx}, 1}$...	$P_{n, 1, \text{xxx}} =$ $P_{n, 1} * FX_{n/\text{xxx}, 1}$
Revaluation 2	$P_{1, 2, \text{xxx}} =$ $P_{1, 2} * FX_{1/\text{xxx}, 2}$	$P_{2, 2, \text{xxx}} =$ $P_{2, 2} * FX_{2/\text{xxx}, 2}$...	$P_{n, 2, \text{xxx}} =$ $P_{n, 2} * FX_{n/\text{xxx}, 2}$
...
...
Revaluation LP - 1	$P_{1, \text{LP} - 1, \text{xxx}} =$ $P_{1, \text{LP} - 1} * FX_{1/\text{xxx}, \text{LP} - 1}$	$P_{2, \text{LP} - 1, \text{xxx}} =$ $P_{2, \text{LP} - 1} * FX_{2/\text{xxx}, \text{LP} - 1}$...	$P_{n, \text{LP} - 1, \text{xxx}} =$ $P_{n, \text{LP} - 1} * FX_{n/\text{xxx}, \text{LP} - 1}$
Revaluation LP	$P_{1, \text{LP}, \text{xxx}} =$ $P_{1, \text{LP}} * FX_{1/\text{xxx}, \text{LP}}$	$P_{2, \text{LP}, \text{xxx}} =$ $P_{2, \text{LP}} * FX_{2/\text{xxx}, \text{LP}}$...	$P_{n, \text{LP}, \text{xxx}} =$ $P_{n, \text{LP}} * FX_{n/\text{xxx}, \text{LP}}$

A product P&L scenario distribution is obtained subtracting current (converted) price from scenario (converted) prices and applying product multiplier:

$$(1) \tilde{P}/L_{\text{product, scenario}} = (\tilde{P}_{\text{product, scenario}} * \tilde{FX}_{\text{scenario}} - P_{\text{product, current}} * FX_{\text{current}}) * \text{multiplier}_{\text{product}}$$

for *option* and cash products,

while subtracting current price from scenario prices, converting and applying product multiplier:

$$(2) \tilde{P}/L_{\text{product, scenario}} = (\tilde{P}_{\text{product, scenario}} - P_{\text{product, current}}) * \tilde{FX}_{\text{scenario}} * \text{multiplier}_{\text{product}}$$

for *futures*.

The difference is due to the fact that *futures* positions are subject to daily posting of *Variation margins* and do not imply any outflows/inflows at trade inception, as opposed to *options* and cash products. This has implications in terms of close-out trades, which in turn result in the above difference in formulas.

The formula for *option* and cash products also applies to physically delivered *futures* that have expired but have not settled yet and to exercised/assigned *options*, with due modifications illustrated here below.

Positions in physically delivered futures that have expired but have not settled yet

$\tilde{P}_{\text{product, scenario}}$ and $P_{\text{product, current}}$ are intended to be those of the underlying asset.

$\text{multiplier}_{\text{product}}$ is that of the *futures*.

Positions in exercised/assigned options

$\tilde{P}_{\text{product, scenario}}$ and $P_{\text{product, current}}$ are intended to be:

- *call options*: $(\tilde{P}_{\text{underlying_asset, scenario}} - K)$ and $(P_{\text{underlying_asset, current}} - K)$;
- *put options*: $(K - \tilde{P}_{\text{underlying_asset, scenario}})$ and $(K - P_{\text{underlying_asset, current}})$.

$multiplier_{product}$ is that of the *option*.

Table 4: Clearing Member's portfolio of products' P&L in clearing currency xxx

Scenario	Product 1 P/L in clearing currency xxx	Product 2 P/L in clearing currency xxx	...	Product n P/L in clearing currency xxx
Revaluation 1	$P/L_{1,1,xxx} = (P_{1,1,xxx} - P_{1,current,xxx}) * multiplier_1$	$P/L_{2,1,xxx} = (P_{2,1,xxx} - P_{2,current,xxx}) * multiplier_2$...	$P/L_{n,1,xxx} = (P_{n,1,xxx} - P_{n,current,xxx}) * multiplier_n$
Revaluation 2	$P/L_{1,2,xxx} = (P_{1,2,xxx} - P_{1,current,xxx}) * multiplier_1$	$P/L_{2,2,xxx} = (P_{2,2,xxx} - P_{2,current,xxx}) * multiplier_2$...	$P/L_{n,2,xxx} = (P_{n,2,xxx} - P_{n,current,xxx}) * multiplier_n$
...
...
Revaluation LP - 1	$P/L_{1,LP-1,xxx} = (P_{1,LP-1,xxx} - P_{1,current,xxx}) * multiplier_1$	$P/L_{2,LP-1,xxx} = (P_{2,LP-1,xxx} - P_{2,current,xxx}) * multiplier_2$...	$P/L_{n,LP-1,xxx} = (P_{n,LP-1,xxx} - P_{n,current,xxx}) * multiplier_n$
Revaluation LP	$P/L_{1,LP,xxx} = (P_{1,LP,xxx} - P_{1,current,xxx}) * multiplier_1$	$P/L_{2,LP,xxx} = (P_{2,LP,xxx} - P_{2,current,xxx}) * multiplier_2$...	$P/L_{n,LP,xxx} = (P_{n,LP,xxx} - P_{n,current,xxx}) * multiplier_n$

A portfolio P&L scenario distribution is instead obtained summing product (converted) P/Ls, applied to position size, in every scenario.

$$(3) \widetilde{P/L}_{portfolio,scenario} = \sum_i \widetilde{P/L}_{product i,scenario} * n_contracts_i$$

If one wants to express losses as positive quantities and profits as negative quantities the number of contracts must be computed subtracting long positions from short positions (S – L).

Table 5: Clearing Member's portfolio's P&L in clearing currency xxx

Scenario	Portfolio P/L in clearing currency xxx
Revaluation 1	$P/L_{1,xxx} = P/L_{1,1,xxx} * n_contracts_1 + P/L_{2,1,xxx} * n_contracts_2 + \dots + P/L_{n,1,xxx} * n_contracts_n$
Revaluation 2	$P/L_{2,xxx} = P/L_{1,2,xxx} * n_contracts_1 + P/L_{2,2,xxx} * n_contracts_2 + \dots + P/L_{n,2,xxx} * n_contracts_n$
...	...
...	...
Revaluation LP - 1	$P/L_{LP-1,xxx} = P/L_{1,LP-1,xxx} * n_contracts_1 + P/L_{2,LP-1,xxx} * n_contracts_2 + \dots$

	$\dots +$ $P/L_{n, LP-1, xxx} * n_contracts_n$
Revaluation LP	$P/L_{LP, xxx} =$ $P/L_{1, LP, xxx} * n_contracts_1 +$ $P/L_{2, LP, xxx} * n_contracts_2 +$ $\dots +$ $P/L_{n, LP, xxx} * n_contracts_n$

The relevant risk measure is finally computed on the portfolio P&L scenario distribution and represents the Clearing Member's portfolio *Initial Margins*. The risk measure can be:

- ES (average of tail observations)/VaR (first observation outside the tail of the distribution);
- with single (only actual values of losses are taken into account)/double (absolute values of both gains and losses are taken into account) tail approach;
- if ES, with equal/different weighting of tail events.

The number of tail observations is a function of the chosen *confidence level* α and *lookback period* LP :

$$(4) \ n_tail_observations = LP * (1 - \alpha)$$

The rounding of the above multiplication is at the nearest integer. If the decimal part of the result is exactly equal to 0.5 the rounding is prudentially down. If the rounding leads to a (rounded) result of 0, 1 value is forced. In that case, the ES will coincide with that tail observation (being the average of just 1 number).

Table 6: Clearing Member's portfolio's risk measure in clearing currency xxx

Portfolio risk measure in clearing currency xxx
$P\&L_{xxx}$ risk measure

1.3 Ordinary and Stressed historical risk measures

The chosen risk measure is based on risk factor returns which are historically simulated. This historical risk measure (i.e. *Initial Margins*) is computed under two different configurations:

- *Ordinary*;
- *Stressed*.

These are two different quantities and are the output of distinct computations. These calculations are executed on the same Clearing Members' portfolios of positions and share the vast majority of methodological details. The fundamental discrepancies are:

- risk factor returns are scaled (filtered according to dynamic volatility) for *Ordinary Initial Margins* calculation purposes only;
- chosen *lookback periods* differ in terms of length and construction methodology;
- (other) input parameters may differ.

Stressed Initial Margins play a countercyclical (APC) role improving stability in margin requirements during stress periods.

1.4 Risk factors impacting financial products

The driving risk factors for *Initial Margins* calculation purposes are:

- prices of cash instruments and values of stock indices (including underlying assets of derivative contracts);
- *nearby* (expiry1, expiry2, ...) *futures* prices in the case of *dividend futures*; warrant, certificate prices (despite derivative or semi-derivative contracts, for materiality reasons);
- (implied) volatility surfaces of assets underlying *option* contracts – pivot *moneyness-time to maturity* coordinates;
- risk-free interest rate curves for relevant product currencies – tenors;
- exchange rates for relevant product currency-allowed clearing currency pairs.

For a given variable the status of ‘risk factor’ implies that its current value undergoes revaluation under a given set of scenarios.

Dividend forecasts and borrow (repo) rates linked to cash instruments/stock indices are examples of variables which are not considered ‘risk factor’. This means that their current values are assumed to stay constant under the same given set of scenarios.

1.5 Scenario (re)valuation of financial products (per position type)

Product scenario prices are calculated by applying *holding period*-returns to its risk factors and revaluating the product.

The revaluation mechanism per position type is described below.

- Cash positions – plus warrant, certificate positions

Current, neutral scenario price is the current price employed for *Mark-to-market Margins* calculation purposes. Revaluation scenario prices are obtained applying price returns to this price, according to what outlined in Subsection 2.2. The profit/loss recorded in that revaluation scenario will be equal to the difference between revaluation scenario price and current, neutral scenario price.

- Positions in physically delivered *futures* that have expired but have not settled yet

Current, neutral scenario price is the current price of the underlying. Revaluation scenario prices are obtained applying price returns to this price, according to what outlined in Subsection 2.2. The profit/loss recorded in that revaluation scenario will be equal to the difference between revaluation scenario price and current, neutral scenario price.

- Positions in exercised/assigned *options*

Current, neutral scenario price is the price of the underlying. Revaluation scenario prices are obtained applying price returns to this price, according to what outlined in Subsection 2.2. The profit/loss recorded in that revaluation scenario will be equal to the difference between revaluation scenario price and current, neutral scenario price.

- Positions in dividend *futures*

Current, neutral scenario price is the current price employed for *Variation Margins* calculation purposes. Revaluation scenario prices are obtained applying price returns of the proper *nearby* to this price, according to what outlined in Subsection 2.22.3. The profit/loss recorded in that revaluation scenario will be equal to the difference between revaluation scenario price and current, neutral scenario price.

- Other derivatives (*futures, option*) positions – apart from warrants, certificates

Current, neutral scenario price is the theoretical (fair, model) price computed employing current risk factor (and other variable) values. Revaluation scenario prices are the theoretical (fair, model) prices computed employing risk factor scenario values, in turn computed according to what outlined in Subsections 2.2, 2.4 and 2.5 (plus other variable current values). The profit/loss recorded in that revaluation scenario will be equal to the difference between revaluation scenario price and current, neutral scenario price. Please refer to Section 5 for derivative pricing formulas.

2 Building of risk factor returns

2.1 Inputs: Parameters

In order to compute returns the input risk factor time series must have length at least equal to the chosen *lookback period* plus the chosen *holding period* (for the relevant *Initial Margins*).

Since a scaling procedure is applied to risk factor returns for *Ordinary Initial Margins* calculation purposes, in that case the input risk factor time series must have length equal to the chosen *lookback period* plus:

- the chosen *holding period*;
- the chosen *scaling window* (employed for seed EWMA volatility calculation purposes).

As for *Stressed Initial Margins*, *lookback period* and *holding period* will be sufficient.

2.2 Cash/underlying prices

The content of this Subsection also applies to warrants, certificates, despite derivative or semi-derivative contracts (for materiality reasons) and the abovementioned small portion of bonds.

For a given day t in the *lookback period*, cash/underlying price returns are computed according to the following formula:

$$(5) \Delta S_t = \ln\left(\frac{S_t}{S_{t-HP}}\right),$$

where:

- S_t is the price at time t ;
- HP is the chosen *holding period*.

When it comes to applying the computed price returns (each of them representing a given scenario) to the current cash/underlying price to obtain scenario prices, the relevant formula is the following:

$$(6) \tilde{S}_t = S_{current} * e^{\Delta S_t},$$

where $S_{current}$ is the current price.

2.3 Dividend futures prices

As mentioned above, *nearby* (expiry1, expiry2, ...) *dividend futures* prices are part of the set of model risk factors.

This means that *dividend futures* are day-by-day tracked as *nearbys* and these *nearby* prices are the elective risk factors for this product class.

For a given *nearby* and a given day t in the *lookback period*, *dividend futures* price returns are computed according to the following formula:

$$(7) \Delta S_t = S_t - S_{t-HP},$$

where:

- S_t is the price at time t ;
- HP is the chosen *holding period*.

When it comes to applying the computed price returns (each of them representing a given scenario) to the current price to obtain scenario prices, the relevant formula is the following:

$$(8) \tilde{S}_t = S_{current} + \Delta S_{t,nearby},$$

where $S_{current}$ is the current price.

It's important to stress that on a given (margin) date a given *dividend futures* must be traced back to a given *nearby* in order to apply formula above.

Conditional on the availability of dividend payment dates, *dividend futures* price returns are set to 0 in case there are no dividend payments left up to the *dividend futures* expiry. For those underlyings with multiple dividend payments over the life of the *dividend futures* price returns are scaled down employing a scaling factor SF equal to

$$1 - \min\left(\frac{\sum_{t_i \in \text{dividend futures period}} \text{paid dividend amount}_i}{\text{dividend futures price}}, 1\right).$$

2.4 Options' underlyings' implied volatilities

As mentioned above, for each asset underlying *option* contracts a fixed set of pivot *moneyness-time to maturity* coordinates on the implied volatility surface represents the set of risk factors. A set of just one pair of pivot coordinates (e.g. *moneyness*: 1 (at-the-money)-*time to maturity*: 0.5Y) would imply that resulting implied volatility surface scenarios would be parallel, following the movements of that pair of pivot coordinates. On the other hand, numerous pairs would capture different aspects of volatility surface movements (slope, curvature) but be computationally unfeasible. A right balance is therefore needed, also having in mind that trading volumes and market maker bid-ask spreads (thus liquidity, thus accuracy of input data) are concentrated around the at-the-money *moneyness* point and decrease almost linearly as *time to maturity* decreases.

For a given day t in the *lookback period*, having implied volatility data stored as *option*-implied volatility pairs the first step is computing *moneyness* $\left(\frac{S_t}{K}\right)$, with S_t : underlying asset price and K : strike price) and *time to maturity* ($T - t$ in year fractions, with T : expiry) (hereinafter, the coordinates) of all available *options*. Then, every pivot *moneyness-time to maturity* coordinate will be matched to the implied volatility of the *option* which minimizes the distance between pivot coordinates and *option* coordinates. The distance equation is the following:



$$(9) \text{ distance} = \sqrt{(\overrightarrow{M_{option}} - \overrightarrow{M_{pivot}})^2 + (\overrightarrow{TTM_{option}} - \overrightarrow{TTM_{pivot}})^2},$$

where:

- M_{option} : *moneyiness* of the *option*;
- $\overrightarrow{M_{option}}$: normalized M_{option} , computed as $\frac{M_{option} - \mu_M}{\sigma_M}$;
- μ_M : average *moneyiness* across available *options*;
- σ_M : *moneyiness* sample standard deviation across available *options*;
- M_{pivot} : pivot *moneyiness*;
- $\overrightarrow{M_{pivot}}$: normalized M_{pivot} , computed as $\frac{M_{pivot} - \mu_M}{\sigma_M}$;
- TTM_{option} : *time to maturity* of the *option*;
- $\overrightarrow{TTM_{option}}$: normalized TTM_{option} , computed as $\frac{TTM_{option} - \mu_{TTM}}{\sigma_{TTM}}$;
- μ_{TTM} : average *time to maturity* across available *options*;
- σ_{TTM} : *time to maturity* sample standard deviation across available *options*;
- TTM_{pivot} : pivot *time to maturity*;
- $\overrightarrow{TTM_{pivot}}$: normalized TTM_{pivot} , computed as $\frac{TTM_{pivot} - \mu_{TTM}}{\sigma_{TTM}}$.

Having all pivot *moneyiness-time to maturity* coordinate implied volatility data and reference *options*, for a given day t in the *lookback period* and pivot *moneyiness-time to maturity* coordinate (M, TTM) , underlying asset implied volatility returns are calculated according to the following formula:

$$(10) \Delta\sigma_{t,M,TTM} = \ln \left(\frac{\sigma_{t, \min_distance_option_t-HP}}{\sigma_{(t-HP),M,TTM}} \right),$$

where:

- σ_t is the implied volatility at time t ;
- $\sigma_{\min_distance_option_t-HP}$ is the implied volatility at time t of the *option* with minimum distance to the pivot coordinate at time $t-HP$ (i.e. whose implied volatility is $\sigma_{(t-HP),M,TTM}$);
- HP is the chosen *holding period*.

Example

For example, suppose we track [*moneyiness: 1-time to maturity: 0.5Y*] as pair of pivot coordinates (among others). Suppose also we are interested in the historical returns in implied volatility for the underlying ISIN IT0000000001, and one of the historical dates is the 06/01/2022. If we assume a *holding period* equal to 3 *business days* the return will be between 03/01/2022 and 06/01/2022. The first input we need is the reference *option* for the date 03/01/2022, that underlying and that pair of pivot coordinates, together with its implied volatility: suppose those are *option* ISIN IT0000000002 and 20%. We then need the implied volatility of the

same ISIN IT0000000002 on the 06/01/2022: suppose that is 30%. The return associated to the 06/01/2022-underlying ISIN IT0000000001-pair of implied volatility pivot coordinates [moneyness: 1-time to maturity: 0.5Y] combination can finally be computed as $\ln\left(\frac{30\%}{20\%}\right)$, i.e. the ratio between the implied volatility on the t date of the *option* which is referenced for the given pair of pivot coordinates at t -HP and its (pivot) value on the t -HP date itself.

When it comes to applying the computed implied volatility returns (each of them representing a given scenario) to the current implied volatility of an *option* to obtain scenario implied volatilities for that *option*, it is important to stress that the implied volatility returns to select will be those among all pivot *moneyness-time to maturity* coordinates minimizing the normalized distance between the *option* current coordinates and the pivot coordinates (see formula (9) above). This time the normalization must be applied employing pivot coordinates' average and standard deviation statistics for *moneyness* and *time to maturity*.

The relevant formula will then be:

$$(11) \tilde{\sigma}_{t,option} = \sigma_{current,option} * e^{\Delta\sigma_{t,M,TTM}},$$

where $\sigma_{current}$ is the current implied volatility.

2.5 Risk-free interest rates

Risk-free interest rates are risk factors which affect the valuation of derivative contracts.

For a given day t in the *lookback period*, product currency (n) and tenor (y), risk-free interest rate returns are computed according to the following formula:

$$(12) \Delta r_{t,n,y} = r_{t,n,y} - r_{(t-HP),n,y},$$

where:

- $r_{t,n,y}$ is the rate at time t ;
- HP is the chosen *holding period*.

When it comes to applying the computed rate returns (each of them representing a given scenario) to the current rate to obtain scenario rates, the relevant formula is the following:

$$(13) \tilde{r}_{t,n,y} = r_{current,n,y} + \Delta r_{t,n,y},$$

where $r_{current}$ is the current rate.

2.6 Foreign exchange rates

Foreign exchange rates are employed to convert product currencies to clearing currency(ies) for margin purposes.

For a given day t in the *lookback period*, foreign exchange rate returns are computed according to the following formula:

$$(14) \Delta FX_t = \ln \left(\frac{FX_t}{FX_{(t-HP)}} \right),$$

where:

- FX_t is the rate at time t ;
- HP is the chosen *holding period*.

When it comes to applying the computed rate returns (each of them representing a given scenario) to the current rate to obtain scenario rates, the relevant formula is the following:

$$(15) \widetilde{FX}_t = FX_{current} * e^{\Delta FX_t},$$

where $FX_{current}$ is the current rate.

3 Scaling of risk factors returns

Scaling of risk factor returns will be applied only for *Ordinary Initial Margins* calculation purposes.

3.1 Inputs: Parameters and risk factor values

The scaling procedure is applied to the risk factor returns computed according to the methodologies described in Section 2. As mentioned above, for scaling purposes the input risk factor time series must have length equal to the chosen *lookback period* (LP) plus:

- the chosen *holding period* (HP);
- the chosen *scaling window* (SW , employed for seed EWMA volatility calculation purposes).

In addition to these, another parameter will be employed in the procedure: the smoothing factor λ , which can take value in the interval $(0,1)$.

The time series of a given input risk factor values can be generalized as follows (sorting: by 'Date', descending):

Table 7: Risk factor value time series

Date	Risk factor value
1	v1
2	v2
...	...
...	...
$LP + SW + HP - 1$	$v_{LP + SW + HP - 1}$
$LP + SW + HP$	$v_{LP + SW + HP}$

3.2 Inputs: (Unscaled) Risk factor returns

The above time series of values can be turned into a time series of (unscaled) returns employing formula (5)(7)(10), (12) or (14) (depending on the risk factor class). This new time series of returns has now length equal to the chosen *lookback period* plus the chosen *scaling window*.

Table 8: Risk factor return time series (1)

Date	Risk factor return
1	r1
2	r2
...	...
...	...
$LP + SW - 1$	$r_{LP + SW - 1}$
$LP + SW$	$r_{LP + SW}$

3.3 EWMA volatility

Given the above time series of returns, for each date t in the *lookback period* it is then necessary to compute a EWMA volatility value. To this aim, a seed volatility value is needed as the EWMA volatility formula is a recursive formula. This seed volatility is computed as the sample standard deviation of the returns in the *scaling window*:

$$(16) \sigma_{seed} = \sqrt{\sum_{i=1}^n \frac{(r_i - \bar{r})^2}{n-1}},$$

where:

- $r_{i=1,2,\dots,n}$ are the returns in the *scaling window*;
- \bar{r} is the average of these returns.

The time series of returns can now be reduced to the *lookback period* time series:

Table 9: Risk factor return time series (2)

Date	Risk factor return
1	r1
2	r2
...	...
...	...
LP - 1	r _{LP-1}
LP	r _{LP}

Each return in the *lookback period* is then paired to a EWMA volatility value. The EWMA volatility formula is the following:

$$(17) \sigma_{EWMA,i} = \sqrt{\lambda * \sigma_{EWMA,i-1}^2 + (1 - \lambda) * r_i^2}.$$

Despite dates in this Section's tables are ordered in descending way (from most recent to oldest), above and following formulas indicate the date preceding a given date i as $i-1$.

The recursiveness of the formula is evident, as $\sigma_{EWMA,i}$ depends on the value of $\sigma_{EWMA,i-1}$. The first $\sigma_{EWMA,i}$ to compute is that associated to the farthest return of the *lookback period* (the 'LP' 'Date' one in the table above). Only in this case a $\sigma_{EWMA,i-1}$ value is not available, and here is why σ_{seed} is needed. Indeed, formula (17) for $i = LP$ becomes:

$$(18) \sigma_{EWMA,LP} = \sqrt{\lambda * \sigma_{seed}^2 + (1 - \lambda) * r_i^2}.$$

Formula (17), which is a variant of the formula $\sigma_{EWMA,i} = \sqrt{\lambda * \sigma_{EWMA,i-1}^2 + (1 - \lambda) * r_{i-1}^2}$, allows to weigh differently the returns based on the current volatility regime.

Table 10: Risk factor return time series with EWMA volatility

Date	Risk factor return	EWMA volatility
1	r1	$\sigma_{EWMA,1} = f(\sigma_{EWMA,2}, r1, \lambda)$
2	r2	$\sigma_{EWMA,2} = f(\sigma_{EWMA,3}, r2, \lambda)$
...
...
LP - 1	r _{LP-1}	$\sigma_{EWMA,LP-1} = f(\sigma_{EWMA,LP}, r_{LP-1}, \lambda)$
LP	r _{LP}	$\sigma_{EWMA,LP} = f(\sigma_{seed}, r_{LP}, \lambda)$

3.4 Scaling factors

EWMA volatilities are employed to compute the scaling factors which will finally be applied to the returns in order to obtain scaled returns. The scaling factor applied to a given return I is computed employing the following formula (mid-volatility approach):

$$(19) \text{scaling_factor}_i = \frac{\sigma_{EWMA,1} + \sigma_{EWMA,i}}{2 * \sigma_{EWMA,i}}$$

where $\sigma_{EWMA,1}$ is the EWMA volatility associated to the most recent return.

Formula (19) is a variant of the full-volatility approach formula $\text{scaling_factor}_i = \frac{\sigma_{EWMA,1}}{\sigma_{EWMA,i}}$.

Table 11: Risk factor return time series with EWMA volatility and scaling factor

Date	Risk factor return	EWMA volatility	Scaling factor
1	r1	$\sigma_{EWMA,1} = f(\sigma_{EWMA,2}, r1, \lambda)$	$sf1 = f(\sigma_{EWMA,1}) = 1$
2	r2	$\sigma_{EWMA,2} = f(\sigma_{EWMA,3}, r2, \lambda)$	$sf2 = f(\sigma_{EWMA,1}, \sigma_{EWMA,2})$
...
...
LP - 1	r _{LP-1}	$\sigma_{EWMA,LP-1} = f(\sigma_{EWMA,LP}, r_{LP-1}, \lambda)$	$sf_{LP-1} = f(\sigma_{EWMA,1}, \sigma_{EWMA,LP-1})$
LP	r _{LP}	$\sigma_{EWMA,LP} = f(\sigma_{seed}, r_{LP}, \lambda)$	$sf_{LP} = f(\sigma_{EWMA,1}, \sigma_{EWMA,LP})$

3.5 Scaled risk factor returns

Scaling factors are applied to returns and scaled returns are retrieved:

$$(20) \text{scaled_risk_factor_return}_i = \text{unscaled_risk_factor_return}_i * \text{scaling_factor}_i.$$

Table 12: Risk factor scaled return time series

Date	Risk factor return	EWMA volatility	Scaling factor	Scaled risk factor return
1	r_1	$\sigma_{EWMA,1} = f(\sigma_{EWMA,2}, r_1, \lambda)$	$sf_1 = f(\sigma_{EWMA,1}) = 1$	$sr_1 = f(r_1, sf_1)$
2	r_2	$\sigma_{EWMA,2} = f(\sigma_{EWMA,3}, r_2, \lambda)$	$sf_2 = f(\sigma_{EWMA,1}, \sigma_{EWMA,2})$	$sr_2 = f(r_2, sf_2)$
...
...
LP - 1	r_{LP-1}	$\sigma_{EWMA,LP-1} = f(\sigma_{EWMA,LP}, r_{LP-1}, \lambda)$	$sf_{LP-1} = f(\sigma_{EWMA,1}, \sigma_{EWMA,LP-1})$	$sr_{LP-1} = f(r_{LP-1}, sf_{LP-1})$
LP	r_{LP}	$\sigma_{EWMA,LP} = f(\sigma_{secd}, r_{LP}, \lambda)$	$sf_{LP} = f(\sigma_{EWMA,1}, \sigma_{EWMA,LP})$	$sr_{LP} = f(r_{LP}, sf_{LP})$

3.6 Scaled risk factor scenario values

Scaled risk factor scenario values are obtained employing the same formulas as unscaled ones (formulas(6), (8), (11), (13) or (15)). The only difference is that scaled risk factor returns are applied to risk factor current values rather than (unscaled) risk factor returns.

4 Missing historical risk factor data

Equities/Equity Derivatives asset class is characterized by the fact that every underlying firm whose value is traded on the market has its own riskiness. Therefore, risk factor amount is huge compared to other asset classes. Furthermore, always compared to other asset classes, these several risk factor time series are not as robust and deep. This is due to the fact that every time series has its own starting and ending point and may change shape and characteristics multiple times. Finally, the liquidity (hence, quantity of stale values) of a risk factor data may be very different from another one's.

Among all of the abovementioned issues, the most relevant one for our purposes is probably the following: one willing to build his/her model leveraging on a (potentially long) set of common, historical dates has to deal with missing data. Therefore, a method for filling data gaps is needed and this method has to be as reliable and flexible as possible.

The chosen method is employing reference sector (if possible and material, e.g. in case of stocks) and market stock index data as fallbacks¹. This means that the first fallback is a sector benchmark, and in case needed the second fallback is the reference market stock index.

Furthermore, the possibility of employing ad-hoc benchmarks is preserved in order to properly tackle (potential) peculiar cases.

A sector is assigned as benchmark according to multiple criteria; for example, some criteria are:

- market capitalization (representativity);
- availability of deep, accurate and liquid data;
- expert judgement.

The assignment is reviewed at discrete time intervals or in any case needed (e.g. delisting of the benchmark).

Examples of sector benchmarks may be ENEL for Italian energy sector, INTESA SANPAOLO for Italian banking sector,

Market stock index risk factor data are usually very deep, accurate and liquid. Furthermore, having a look at the concentration of trading volumes, it is also evident that these data can be considered to be sufficiently (if not very) representative.

An example of reference market stock index is the FTSE MIB index for financial products traded on the Italian market.

¹ For the abovementioned small portion of bonds the dynamics observed on their relevant sector curve is employed.



This method is very robust and easy to implement. Most of the work is indeed static data work.

Therefore, when needed, cash/underlying price and volatility surface return data will be filled with sector/market stock index benchmark value and volatility surface variations, respectively.

(Nearby) Dividend futures price return data will rely only on market stock index benchmark *dividend futures* price variations.

5 Derivatives pricing formulas

5.1 *Futures*

Pricing is a function of (ignoring expiry T):

$$(21) P_s = f(S_s, rfr_s, (dividend_{current}), (repo_{current})),$$

where:

- P_s : scenario price (including current, neutral scenario);
- S_s : scenario underlying asset price/value (including current, neutral scenario);
- rfr_s : scenario risk-free curve (including current, neutral scenario);
- $dividend_{current}$: (potentially) current, neutral scenario cash dividends (with ex-dates);
- $repo_{current}$: (potentially) current, neutral scenario underlying asset borrow (repo) curve.

Assuming the availability of risk-free interest rate curves, if *futures* expiry date and/or (potential) cash instrument/stock index cash dividend ex-date(s) don't match a tracked tenor of the relevant curve, linear interpolation (between two tenors)/flat extrapolation (before first tenor and after last tenor) techniques must be applied to tracked tenors in order to obtain current rates and scenario rates corresponding to the abovementioned dates.

If cash instrument/stock index borrow (repo) rates are only available at *futures* expiry dates, borrow (repo) rates corresponding to (potential) cash instrument/stock index cash dividend ex-date(s) are easily computable employing those data and linear interpolation (between two expiries)/flat extrapolation (before first expiry and after last expiry) techniques. Otherwise, in case of availability of cash instrument/stock index borrow (repo) curves and if *futures* expiry date and/or (potential) cash instrument/stock index cash dividend ex-date(s) don't match a tracked tenor of the relevant curve, linear interpolation (between two tenors)/flat extrapolation (before first tenor and after last tenor) techniques must be applied to tracked tenors in order to obtain current rates corresponding to the abovementioned dates.

5.1.1 *Stock and stock index futures*

Discrete dividends are employed (when existing):

$$(22) P_s = (S_s - \sum_{i | t_i \leq T} dividend_i * e^{-(rfr_{s,t_i} - repo_{t_i}) * t_i}) * e^{(rfr_{s,T} - repo_T) * T},$$

where:

- P_s : scenario price (including current, neutral scenario);
- S_s : scenario underlying asset price/value (including current, neutral scenario);

- t : cash dividend ex-date;
- T : *futures* expiry date;
- $dividend_{current}$: (potentially) current, neutral scenario cash dividends;
- rfr_s : scenario risk-free curve (including current, neutral scenario);
- $repo_{current}$: (potentially) current, neutral scenario underlying asset borrow (repo) curve.

5.1.2 Stock index total return futures

$$(23) P_S = S_S + CDI - CFI + SPREAD = SDI_S + SPREAD,$$

where:

- P_S : scenario price (including current, neutral scenario);
- S_S : scenario underlying index value (including current, neutral scenario);
- CDI : Cumulative (i.e. without annual reset) Dividend Index price linked to the underlying index;
- CFI : Cumulative Funding (i.e. accrued interest without reset) Index price linked to the underlying index;
- SDI : Synthetic Dividend Index price linked to the underlying index;
- $SPREAD$: maturity-dependent premium/discount spread above/below risk-free rate.

5.2 Option

Pricing is a function of (ignoring expiry T and strike price K):

$$(24) P_S = f(S_S, rfr_s, \sigma_s, (dividend_{current}), (repo_{current})),$$

where:

- P_S : scenario price (including current, neutral scenario);
- S_S : scenario underlying asset price (including current, neutral scenario);
- rfr_s : scenario risk-free curve (including current, neutral scenario);
- σ_s : scenario *option* implied volatility (including current, neutral scenario)
- $dividend_{current}$: (potentially) current, neutral scenario cash dividends (with ex-dates);
- $repo_{current}$: (potentially) current, neutral scenario underlying asset borrow (repo) curve.

See what highlighted above (for *futures* products) on risk-free interest rate curve and borrow (repo) curve interpolation/extrapolation.



As stressed above, a given *option* will be revalued applying to its current implied volatility the implied volatility returns of the pivot *moneyness-time to maturity* coordinate minimizing the distance between the *option* current coordinates and the pivot coordinates.

5.2.1 Stock and stock index option – European

Black-Scholes model with discrete dividends (when existing) is employed:

$$(25) P_{call,s} = (S_s - \sum_{i|t_i \leq T} dividend_i * e^{-(rfr_{s,t_i} - repo_{t_i}) * t_i}) * e^{-repo_T * T} * N(d_1) - K * e^{-rfr_{s,T} * T} * N(d_2),$$

$$(26) P_{put,s} = K * e^{-rfr_{s,T} * T} * N(-d_2) - (S_s - \sum_{i|t_i \leq T} dividend_i * e^{-(rfr_{s,t_i} - repo_{t_i}) * t_i}) * e^{-repo_T * T} * N(-d_1),$$

$$(27) d_1 = \frac{\ln\left(\frac{(S_s - \sum_{i|t_i \leq T} dividend_i * e^{-(rfr_{s,t_i} - repo_{t_i}) * t_i})}{K}\right) + (rfr_{s,T} - repo_T + \frac{\sigma_s^2}{2}) * T}{\sigma_s * \sqrt{T}},$$

$$(28) d_2 = d_1 - \sigma_s * \sqrt{T},$$

where:

- $P_{call/put,s}$: scenario price (including current, neutral scenario);
- S_s : scenario underlying asset price/value (including current, neutral scenario);
- t : cash dividend ex-date;
- T : *option* expiry date;
- K : *option* strike price;
- σ_s : scenario *option* implied volatility (including current, neutral scenario);
- $dividend_{current}$: (potentially) current, neutral scenario cash dividends;
- rfr_s : scenario risk-free curve (including current, neutral scenario);
- $repo_{current}$: (potentially) current, neutral scenario underlying asset borrow (repo) curve.

5.2.2 Stock and ETF option – American

The binomial tree (lattice) numerical method is employed for pricing options with American exercise-style².

Also in these case, discrete dividends and repo rates are (potential) inputs of the model.

5.3 Rights issue

Pricing reflects the intrinsic value of a call *option* rights issues resemble, this way being consistent with the non-linear nature of their payoff:

² The adopted approach is described in the book of John C. Hull “Options, Futures and Other Derivatives”, 6th edition.

$$(29) P_s = \max((S_s - K) * ER; 0).$$

where:

- P_s : scenario price (including current, neutral scenario);
- S_s : scenario underlying price (including current, neutral scenario);
- K : strike price;
- ER : exchange ratio.

5.4 *Dividend futures*

As mentioned in Subsection 2.3, *nearby* (expiry1, expiry2, ...) *dividend futures* prices are the elective risk factors for the *dividend futures* product class. This implies that their revaluation does not require any pricing formula.

Instead, it follows the methodology outlined in Subsection 2.3. A given *dividend futures* must be first traced back to a given *nearby*.

The current, neutral scenario price will be the *dividend futures* price employed for *Variation Margins* calculation purposes.

Revaluation scenario prices will instead be computed according to formula(8):

$$(30) P_s = P_{current} + \Delta P_{s,nearby}.$$

where:

- $P_{current}$: current, neutral scenario price (*dividend futures* price employed for *Variation Margins* calculation purposes);
- $\Delta P_{s,nearby}$: scenario return of the relevant *dividend futures nearby*.

6 Margined portfolios

The general rule is that margined portfolios are composed by both cash and derivative positions (representing a given Clearing Member's account).

However, the option of separate margin calculations for cash and derivative products is provided to the Clearing Members, upon request. In that case, two different sub-portfolios are created and two different risk measures are computed, without considering any (potential) diversification benefits between cash and derivatives.

The *Initial Margins* calculation description in Subsection 1 is valid also in case of separate margining up to Table 4 (included). Then, two distinct but parallel and similar sets of calculation steps are followed, one per sub-portfolio.

A sub-portfolio P&L scenario distribution is obtained summing product (converted) P/Ls, applied to position size, in every scenario. It is intended that products must belong to the given sub-portfolio.

$$(31) \widetilde{P/L}_{cash_sub-portfolio,scenario} = \sum_i | i \in cash_sub-portfolio \widetilde{P/L}_{product\ i,scenario} * n_contracts_i$$

$$(32) \widetilde{P/L}_{derivative_sub-portfolio,scenario} = \sum_j | j \in derivative_sub-portfolio \widetilde{P/L}_{product\ j,scenario} * n_contracts_j$$

Table 13: Clearing Member's sub-portfolio's P&L in clearing currency xxx

Scenario	Cash sub-portfolio P/L in clearing currency xxx	Derivative sub-portfolio P/L in clearing currency xxx
Revaluation 1	$\begin{aligned} &P/L_{cash, 1, xxx} = \\ &P/L_{1, 1, xxx} * n_contracts_1 + \\ &P/L_{2, 1, xxx} * n_contracts_2 + \\ &\dots + \\ &P/L_{cn, 1, xxx} * n_contracts_{cn} \end{aligned}$	$\begin{aligned} &P/L_{der, 1, xxx} = \\ &P/L_{1, 1, xxx} * n_contracts_1 + \\ &P/L_{2, 1, xxx} * n_contracts_2 + \\ &\dots + \\ &P/L_{dn, 1, xxx} * n_contracts_{dn} \end{aligned}$
Revaluation 2	$\begin{aligned} &P/L_{cash, 2, xxx} = \\ &P/L_{1, 2, xxx} * n_contracts_1 + \\ &P/L_{2, 2, xxx} * n_contracts_2 + \\ &\dots + \\ &P/L_{cn, 2, xxx} * n_contracts_{cn} \end{aligned}$	$\begin{aligned} &P/L_{der, 2, xxx} = \\ &P/L_{1, 2, xxx} * n_contracts_1 + \\ &P/L_{2, 2, xxx} * n_contracts_2 + \\ &\dots + \\ &P/L_{dn, 2, xxx} * n_contracts_{dn} \end{aligned}$
...
...
Revaluation LP - 1	$\begin{aligned} &P/L_{cash, LP-1, xxx} = \\ &P/L_{1, LP-1, xxx} * n_contracts_1 + \\ &P/L_{2, LP-1, xxx} * n_contracts_2 + \\ &\dots + \\ &P/L_{cn, LP-1, xxx} * n_contracts_{cn} \end{aligned}$	$\begin{aligned} &P/L_{der, LP-1, xxx} = \\ &P/L_{1, LP-1, xxx} * n_contracts_1 + \\ &P/L_{2, LP-1, xxx} * n_contracts_2 + \\ &\dots + \\ &P/L_{dn, LP-1, xxx} * n_contracts_{dn} \end{aligned}$
Revaluation LP	$\begin{aligned} &P/L_{cash, LP, xxx} = \\ &P/L_{1, LP, xxx} * n_contracts_1 + \\ &P/L_{2, LP, xxx} * n_contracts_2 + \\ &\dots + \\ &P/L_{cn, LP, xxx} * n_contracts_{cn} \end{aligned}$	$\begin{aligned} &P/L_{der, LP, xxx} = \\ &P/L_{1, LP, xxx} * n_contracts_1 + \\ &P/L_{2, LP, xxx} * n_contracts_2 + \\ &\dots + \\ &P/L_{dn, LP, xxx} * n_contracts_{dn} \end{aligned}$



with cm : number of cash products in the sub-portfolio; dm : number of derivative products in the sub-portfolio.

The relevant risk measures are finally computed on the two sub-portfolio P&L scenario distributions and represent the Clearing Member's sub-portfolio *Initial Margins*.

Table 14: Clearing Member's sub-portfolio's risk measure in clearing currency xxx

Cash sub-portfolio risk measure in clearing currency xxx	Derivative sub-portfolio risk measure in clearing currency xxx
P&L _{cash, xxx} risk measure	P&L _{der, xxx} risk measure

7 Margins on fail positions

Margins on fail positions are calculated using the same methodology as ‘non-fail’ positions. No netting benefits between these two kinds of positions are allowed, as they are kept segregated for all margining purposes. Under no circumstances margin credits on fail (‘non-fail’) positions may compensate margin debits on ‘non-fail’ (fail) positions. Therefore, margins on fail positions are separately calculated and summed, if a debt, to margins on ‘non-fail’ positions. If the total margin requirement on fail positions is a credit, it is set to 0.